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TIME DISPERSIVE EFFECTS IN ULTRASHORT LASER-TISSUE INTERACTIONS

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Abstract

The relative magnitude of time-dispersion in laser-tissue interactions is proportional to the ratio of the radial width of the pulse to its temporal width and to the group velocity dispersion. The dependence of time dispersion effects on its sign and magnitude is discussed.

1 Introduction

The use of ultrashort laser pulses in medical applications offers the possibility for reaching extremely high intensities with relatively low energies and enhanced localizability. This is extremely important in eye surgery when high intensity pulses are used to disrupt tissue, since nearby sensitive structures can be damaged by the shock and acoustic waves resulting from the optical breakdown (ionization of the target medium). While the physical phenomena associated with optical breakdown has lead to intense experimental research, less attention has been devoted to the pulse propagation before optical breakdown occurs.

Although it is well known that the propagation of a laser pulse through aqueous media is described by a nonlinear Schrödinger equation for the pulse envelope, the exact equation that applies to ultrashort pulses is still part of the research problem (e.g. Powell et al., 1993). In this study we focus on the role of time dispersion during the propagation. Although time dispersion has often been assumed to be negligible, this assumption needs to be rechecked for today's ultrashort pulses since time-dispersion is inversely proportional to the pulse duration. Indeed, experimental evidence suggests that femto-second pulses behave differently from longer pulses: They have a lower retinal injury energy threshold (Birngruber et al., 1987) and they seem to resist self-focusing (Strickland and Corkum, 1991). Differences have also been observed between pico-second and nano-second laser tissue interactions (e.g. Vogel et al., 1994, Zysset et al., 1989).

In the following we will analyze the effects of time dispersion on the propagation. Since the model for the pulse propagation is nonlinear, time dispersion effects cannot be analyzed separately and the interaction between the Kerr nonlinearity, radial dispersion and time dispersion has to be considered.

2 Time Dispersion Sign and Magnitude

The propagation of a laser pulse through an aqueous media is described by the nonlinear Schrödinger equation:

$$i\frac{\partial A}{\partial z}+\frac{\partial^2 A}{\partial r^2}+\frac{1}{r}\frac{\partial A}{\partial r}-\gamma\frac{\partial^2 A}{\partial \tau^2}+\kappa|A|^2A=0 \ , \quad (1)$$

where $A(\tau, r, z)$ is the envelope of the electric field of the pulse, $\tau = t - z/c_g$ is the retarded time in a frame moving with the pulse group velocity c_g, z is the axial coordinate in the direction of propagation and $r = \sqrt{x^2+y^2}$ is the radial coordinate in the transverse plane. The nondimensional variables τ, r and z are measured in units of the pulse initial temporal width T_0, the pulse initial radial width a_0 and the diffraction length $L_{diff} = \beta_0 a_0^2$, respectively, where β_0 is the pulse wavenumber. The relative magnitude of the nonlinearity is $\kappa|A|^2$, where

$$\kappa = 4a_0^2\beta_0^2\frac{n_2}{n_0} \ ,$$

n_0 is the linear index of refraction of water and n_2 is the Kerr coefficient. The time dispersion parameter is

$$\gamma = \left(\frac{a_0}{T_0}\right)^2 \left[\beta\frac{\partial^2\beta}{\partial\omega^2}\right]_{\omega_0}$$

where

$$\beta = \frac{\omega n_0(\omega)}{c} \ ,$$

c is the speed of light in vacuum and ω_0 is the pulse frequency. The expression for γ can be also written as the product of two nondimensional parameters:

$$\gamma \sim RG \ , \ R = \left(\frac{a_0}{cT_0}\right)^2 \ , \ G = n_0\omega_0 \left.\frac{\partial^2(\omega n_0)}{\partial\omega^2}\right]_{\omega_0}$$

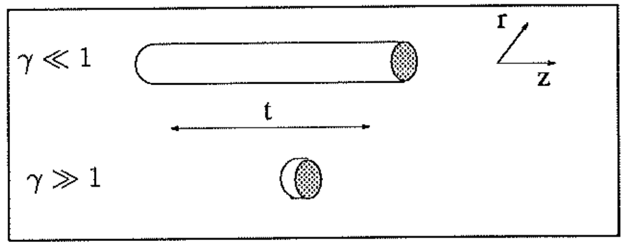

Figure 1: A cigar-like pulse (top) and a disc-like pulse (bottom)

Since cT_0 is the initial temporal pulse width, R is small for pulses that are 'cigar-like' (long and narrow) but is large for 'disc-like' (short and wide) pulses. The value of G depends on the material GVD at the pulse frequency and it determines whether time dispersion is normal ($\gamma > 0$) or anomalous ($\gamma < 0$).

In the visible regime the value of n_0 for water is almost constant. Therefore, $|G| \ll 1$ and time dispersion magnitude can become comparable to radial dispersion only for disc-like pulses.

3 Analysis

We will begin with a short review of the mathematical theory for the interaction of dispersion (radial and/or temporal) with Kerr nonlinearity, as described by the canonical equation

$$i\frac{\partial A}{\partial z} + \alpha\,\Delta_\perp A + \kappa|A|^2A = 0 \ ,$$

where α is a constant and

$$\Delta_\perp = \frac{\partial^2}{\partial x_1^2} + \cdots + \frac{\partial^2}{\partial x_d^2}$$

is the d dimensional transverse Laplacian. The most important parameter is the relative signs of dispersion and the nonlinearity. While dispersion leads to broadening, the nonlinearity will be defocusing when they are of the opposite sign and focusing when they are of the same sign. In the focusing case the interaction depends also on the transverse dimension d. In one dimension the focusing nonlinearity is always balanced by dispersion (they may even balance each other exactly, resulting in soliton propagation). However, in higher dimensions the focusing nonlinearity can become unbalanced by dispersion, resulting in catastrophic self-focusing. In dimension $d = 2$ a necessary condition for collapse is that the pulse power is above a critical value.

3.1 $|\gamma| \gg 1$

In the case of large time dispersion equation (1) can be simplified:

$$i\frac{\partial A}{\partial z} - \gamma\frac{\partial^2 A}{\partial \tau^2} + \kappa|A|^2 A = 0 \qquad (2)$$

This equation also describes the propagation of laser pulses in optical fibers. In equation (2) anomalous time dispersion slows temporal broadening while normal time dispersion enhances it, with the exact dynamics depending on the ratio γ/κ and on the initial pulse form (Agrawal, 1989). If the initial profile is an unchirped Gaussian, then the rms temporal width broadening factor can be approximated by (Potasek et al., 1986):

$$\frac{T}{T_0} \sim \sqrt{1 + 2\sqrt{2}\gamma\phi_m z^2 + \left[1 + \frac{4}{3\sqrt{3}}\phi_m^2 z^2\right](2\gamma z)^2}$$

$$\phi_m = \kappa|A(0,0)|^2 \ , \qquad (3)$$

the approximation being valid for $z\phi_m < 1$. Note that in the case of normal time dispersion the temporal broadening of the pulse is accompanied by a reduction of its power. This may explain why ultrashort pulses in normally dispersive media seem to resist self-focusing.

3.2 $|\gamma| \sim 1$

In the anomalous time dispersion regime when time and radial dispersion are of comparable magnitudes, the propagation dynamics is determined by the interplay between the focusing nonlinearity and the 3D (i.e. (x, y, t)) defocusing Laplacian. This case is supercritical for self-focusing and the temporal compression will enhance the pulse collapse.

The qualitative picture is less clear in the case of normal time dispersion. Preliminary results suggest that in this case, at least initially, the pulse will broaden temporally while compressing radially. Therefore ,radial collapse will be arrested for pulses with power below and even somewhat above the critical one for 2D self-focusing. However, if the pulse is sufficiently intense, the combined effect of temporal broadening and radial collapse means that the effective value of γ decreases. Therefore, the advanced stages of the propagation will be similar to the case $\gamma \ll 1$.

3.3 $|\gamma| \ll 1$

When time dispersion is small compared with radial dispersion and the Kerr nonlinearity, its effect can be expected to be negligible. Nevertheless, if the pulse is undergoing 2D self-focusing, the focusing nonlinearity and the radial dispersion almost completely balance each other and even small time dispersion can have a large effect. In the case of anomalous time dispersion the qualitative dynamics is 3D self-focusing, with the small temporal compression enhancing the collapse. However, when time dispersion is normal the dynamics is much more complex and new phenomena occur. The most pronounced effects that have been observed in numerical simulations of equation (1) with normal time dispersion are a delay in the onset of catastrophic self-focusing and the temporal splitting of the pulse into two peaks (Chernev and Petrov, 1992; Fibich et al., 1995; Luther et al., 1994; Rothenberg, 1992; Zharova et al., 1986). Additional understanding of these phenomena can be gained from a new system of equations that reduces the self-focusing dynamics to the (τ, z) plane using radially-averaged quantities (Fibich et al., 1995):

$$\frac{\partial N}{\partial z} = 2\gamma N_c \frac{\partial^2 \zeta}{\partial \tau^2} \tag{4}$$

$$a^3 \frac{\partial^2 a}{\partial z^2} = -\frac{N - N_c}{M} \tag{5}$$

$$\frac{\partial \zeta}{\partial z} = \frac{1}{a^2} \tag{6}$$

Here $a(\tau, z)$ corresponds to the radial width of the pulse (and is also inversely proportional to its intensity $|A(\tau, 0, z)|$), $N(\tau, z)$ is the pulse power and $\zeta(\tau, z)$ is the local 'time', all quantities corresponding to the r plane $\tau = constant$. The constant $N_c \cong 11.7$ is the critical power for self-focusing and $M \cong 3.46$.

A detailed derivation, study and numerical confirmation of (4–6) is given in Fibich et al. (1995). In the following we will outline the self-focusing dynamics based on (4–6). Initially, time dispersion is negligible and the left side of equation (4) can be set to zero. Each τ cross section would focus independently according to equation (5), following the adiabatic law (Fibich, 1995; Malkin, 1993):

$$a \sim \sqrt{2\left(Z_c(\tau) - z\right)\sqrt{\frac{N_0(\tau) - N_c}{M}}}$$

where $Z_c(\tau) \sim 0.5 M a_0^2 N_0^{-1/2}$ is the location of blowup in the absence of time dispersion. As a result, the temporal gradients will increase and power would radiate away from the fastest focusing cross section (equation 4). Careful analysis shows that the focusing is arrested in the near vicinity of the initial peak but continues elsewhere. Hence, two new peaks are formed which continue to focus without splitting again.

4 Conclusions

Femtosecond pulses as well as sufficiently 'wide' picosecond pulses are disc-like. This work suggests that time dispersion may play an important role in the propagation of these disc-like pulses. Physical experiments can determine if disc-like pulses in normally dispersive media become elongated and have less power as they propagate according to equation (2) or (3). In the case of small normal time dispersion it needs to be determined experimentally whether it increases the critical power for nonlinear self-focusing, delays its onset and leads to temporal peak splitting. When time dispersion effects are important, this may influence the choice of the desired wavelength, the key issue being whether for the specific application it is desirable to operate in the normal or anomalous regime.

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