Optimized Conformal Parameterizations of Cortical Surfaces Using Shape Based Landmark Matching

Lok Ming Lui, Sheshadri Thiruvenkadam, Yalin Wang, Paul Thompson and Tony Chan

Abstract

In this work, we find *meaningful* parameterizations of cortical surfaces utilizing prior anatomical information in the form of anatomical landmarks (sulci curves) on the surfaces. Specifically we generate close to conformal parametrizations that also give a *shape-based* correspondence between the landmark curves. We propose a variational energy that measures the harmonic energy of the parameterization *maps*, and the shape dissimilarity between mapped points on the landmark curves. The novelty is that the computed maps are guaranteed to give a *shape-based* diffeomorphism between the landmark curves. We achieve this by intrinsically modelling our search space of maps as flows of smooth vector fields that do not flow across the landmark curves, and by using the local surface geometry on the curves to define a shape measure. Such parameterizations ensure consistent correspondence between anatomical features, ensuring correct averaging and comparison of data across subjects. The utility of our model is demonstrated in experiments on cortical surfaces with landmarks delineated, which show that our computed maps give a shape-based alignment of the sulcal curves without significantly impairing conformality.

I. Introduction

Parametrization of the cortical surface is a key problem in brain mapping research. Applications include the registration of functional activation data across subjects, statistical shape analysis, morphometry, and processing of signals on brain surfaces (e.g., denoising or filtering). Applications that compare surface data often make use of surface *diffeomorphisms* that result from parameterization. For the above diffeomorphisms to map data consistently across surfaces, parametrizations are required that preserves the original surface geometry as much as possible. Parameterizations should also be chosen so that the resulting diffeomorphisms between surfaces align key anatomical features consistently. This kind of parameterizations with good anatomical features alignment is particularly important for brain disease analysis such as building the average brain shape. This is advantageous as the surface average of many subjects would retain features that consistently occur on sulci, while uniform speed parameterizations may cause these features to cancel out. For illustration, please see Figure 1.

Conformal mapping [1], [2] is particularly convenient for genus-zero cortical surface models since it gives a parameterization without angular distortions, and comes with computational advantages when solving PDEs on surfaces using grid-based and metric-based computations [3]. However, the above parameterization is not guaranteed to map anatomical features, such as sulcal landmarks, consistently from subject to subject [2], [4].

Landmark-based *diffeomorphisms* [4], [5], [6], [7], [8], [9] are often used to compute, or adjust, cortical surface parameterizations. Similarly to the above works, given two cortical surfaces with anatomical landmarks (sulci curves), we want to find close to conformal parameterizations for the surfaces driven by shape based correspondences (*registration*) between the curves. Our work has three main contributions; first, the surface diffeomorphism resulting from our parameterization maps the sulcal curves *exactly*; second, the correspondence is shape based, i.e., maps similarly-shaped segments of sulcal curves to each other; finally, the conformality of the surface parameterizations is preserved to the greatest possible extent.

Optimization of surface diffeomorphisms by landmark matching has been studied intensively. Gu et al. [2] optimized the conformal parametrization by composing an optimal Mӧbius transformation so that it minimizes a landmark mismatch energy. The resulting parameterization remains conformal. Glaunes et al.[6] proposed to generate large deformation diffeomorphisms of the sphere onto itself, given the displacements of a finite set of template landmarks. The diffeomorphism obtained can match landmark features well, but it is, in general, not a conformal mapping, which can be advantageous for solving PDEs on the resulting grids. Leow et al.[7] proposed a level-set based approach for matching different types of features, including points and 2D or 3D curves represented as implicit functions.

Tosun et al. [8] proposed a more automated mapping technique that results in good sulcal alignment across subjects, by combining parametric relaxation, iterative closest point registration and inverse stereographic projection. Wang et al. [4] proposed an energy that computes maps that are close to conformal and also driven by a landmark matching term that measures the Euclidean distance between the specified landmarks.

Many of the above methods e.g. [4], [6] require corresponding landmark points on the surfaces to be labeled in advance. Secondly, the landmark match measures used above are based on Euclidean distance, or overlap of level set functions representing the landmarks, and do not use shape information to guide correspondences of features within curves. So, the resulting correspondences would be unreliable in the case of landmark curves that differ by non-rigid deformations. Finally, constraining the surface diffeomorphism to exactly align the landmark curves during minimization is difficult, e.g. [4], [8].

To resolve the above issues, we propose a method to optimize the conformal parameterization of the surfaces while non-rigidly registering the landmark curves. Specifically, we formulate our problem as a variational energy defined on a search space of diffeomorphisms generated as flows of smooth vector fields. The vector fields are restricted only to those that *do not flow* across the landmark curves (to enforce exact landmark correspondence). Our energy has 2 terms: (1) a shape term to map similar shaped segments of the landmark curves to each other, and (2) a harmonic energy term to optimize the conformality of the parametrization maps.

II. MODEL

Given two cortical surfaces M_1 and M_2, with sulcal landmark curves $\hat{C}_1$ and $\hat{C}_2$ labeled on them. The curves $\hat{C}_i$ have the same topology relative to M_i. These landmarks curves can be detected automatically by the automatic landmark tracking technique introduced by Lui et al. [10]. Here, we want to find diffeomorphisms $\hat{f}_1 : \Omega \subset \Re^2 \to M_1$, $\hat{f}_2 : \Omega \to M_2$ such that $\hat{f}_2$ o $\hat{f}_1^{-1}|_{\hat{C}_1}$ is a shape based diffeomorphism onto $\hat{C}_2$, i.e $\hat{f}_2$ o $\hat{f}_1^{-1}$ maps *similarly shaped* segments of $\hat{C}_1$ and $\hat{C}_2$ to each other. Also we want $\hat{f}_i$ to be as conformal as possible.

To simplify our computations, M_i are firstly conformally parameterized onto the conformal parameter domain D_i. Assume that $\hat{C}_i$ are mapped to C_i on the parameter domain D_i. Thus, our problem is reduced to the 2D problem of finding diffeomorphism $\tilde{f}_i : \Omega \to D_i$ such that $\tilde{f}_2$ o $\tilde{f}_1^{-1}|_{C_1} = C_2$ is a shape-based diffeomorphism onto C_2. We propose our problem as the minimization of a variational energy with respect to diffeomorphisms $\tilde{f}_i : \Omega \to D_i$, subject to the correspondence constraint $\tilde{f}_2$ o $\tilde{f}_1^{-1}(C_1) = C_2$. The energy consists of two terms. The first term measures the harmonic energy of the maps $\tilde{f}_i$, and the second term measures the shape dissimilarity between C_1 and $\tilde{f}_2$ o $\tilde{f}_1^{-1}(C_1)$.

To handle the above correspondence constraint, we move all our computations to the parameter domain Ω using initial diffeomorphisms $f_{0,i} : \Omega \to D_i$. Let $C \subset \Omega$ be a topological representative of C_i, with $f_{0,i}(C) = C_i$. With the above framework, the energy is formulated over Ω, and the search space of diffeomorphisms $\tilde{f}_i : \Omega \to D_i$, subject to $\tilde{f}_2$ o $\tilde{f}_1^{-1}(C_1) = C_2$, can be constructed as time-1 flows of smooth vector fields on Ω that do not flow across C. For the shape term, we measure the shape dissimilarity between the corresponding landmarks which minimizes the difference in *geodesic curvatures* on the corresponding pairs of points on C_1 and C_2. We discuss the details in the following sections.

A. Formulation

The initial diffeomorphisms $f_{0,i}$ give us a convenient way to perform our computations on the domain Ω. Diffeomorphisms $\tilde{f}_i : \Omega \to D_i$ with $\tilde{f}_2$ o $\tilde{f}_1^{-1}(C_1) = C_2$ can be realized through unique diffeomorphisms $f_i : \Omega \to \Omega$ with $f_i(C) = C$, satisfying $\tilde{f}_i = f_{0,i}$ o f_i (Fig. 2(left)). Thus we formulate our problem as the minimization of the following energy over diffeomorphisms $f_i : \Omega \to \Omega$ with $f_i(C) = C$. Denote $\tilde{f}_i = f_{0,i}$ o f_i, $F = [\tilde{f}_1, \tilde{f}_2]$,

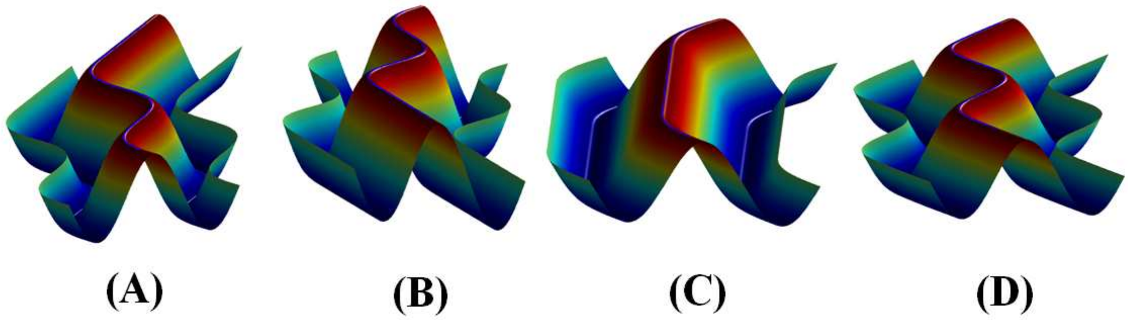

Fig. 1. (A) and (B) shows two surfaces. (C) shows the averaging result of the two surface with arc-length correspondence between landmarks curves. Note that the shape of the landmarks is averaged out and cannot be preserved. (D) shows the averaging result with shape correspondence between landmark curves. Note that the shape of the landmark curve is well preserved.

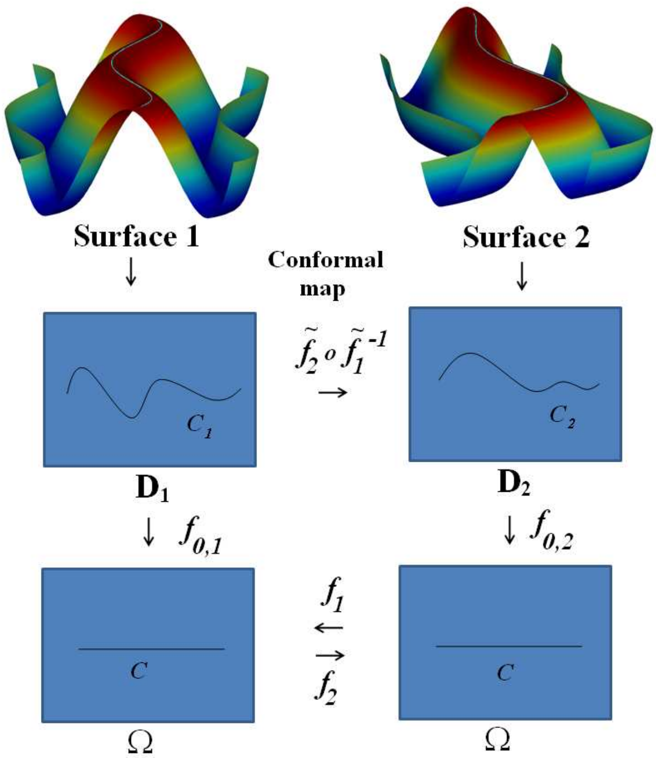

Fig. 2. The figure show the framework of our algorithm.

$$E[f_1, f_2] = \int_\Omega |\nabla \tilde{f}_1|^2 \; + |\nabla \tilde{f}_2|^2 \; dx + \lambda \int_C \left(\kappa_1(\tilde{f}_1) - \kappa_2(\tilde{f}_2)\right)^2 \, |F_x \wedge F_y| \; ds \tag{1}$$

The first term is the harmonic energy of $\tilde{f}_i$. The second term is a *symmetric* shape term defined as an arc length integral over $F(C)$, similar to Thiruvenkadam et al. [11]. Here, the shape measure $\kappa_i(p_i)$ is determined by the geodesic curvature of M_i corresponding to the point p_i. Defining the symmetric shape measure over $F(C)$ makes the term independent of the choice of the initial maps $f_{0,i}$, and also avoids local minima problems that occur while matching flat curve segments.

In the above energy, using a search space of diffeomorphisms $f_i : \Omega \to \Omega$, and then imposing $f_i(C) = C$ as a constraint during minimization is difficult. Hence we propose a method to directly consider a *reduced*

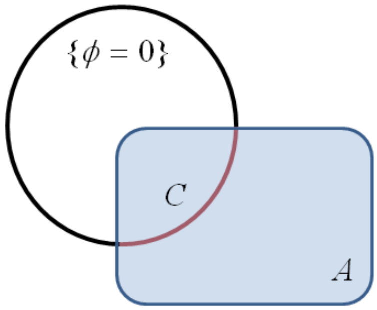

Fig. 3. The figure shows the level set representation for C (Brown open curve), $C = \{\phi = 0\} \cap A$. A is the shaded region, $\{\phi = 0\}$ is the circle.

search space of diffeomorphisms $f_i : \Omega \to \Omega$ that satisfy $f_i(C) = C$.

B. Level Set Representation for C

Since we are dealing with the sulcal curves as our landmarks, we assume that $C = \cup_{k=1}^{N} \Gamma_k$, a union of open curves $\Gamma_k \subset \Omega$. We represent C implicitly in level set form to be able to write the second integral in energy (1) with respect to x. Being the union of open curves, C can be represented as the intersection of the 0-level set of a signed distance function ϕ, and a region A (Fig. 2(right)). Then the arc length integral of C becomes

$$\int_C ds = \int_\Omega \chi_A \, |\nabla H(\phi)| \, dx,$$

where $H(t)$ is a regularized version of the Heaviside function.

C. Modelling the Search Space for f_i

To construct an appropriate search space for f_i, we consider smooth vector fields, $\vec{X}_i = a_i \frac{\partial}{\partial x} + b_i \frac{\partial}{\partial y}$, where $a_i, b_i : \Omega \to \Re$ are C^1 functions with compact support. Then the flow of $\vec{X}_i$, $\Phi^{\vec{X}_i}(\mathbf{x}, t)$ is given by the differential equation,

$$\frac{\partial \Phi^{\vec{X}_i}}{\partial t}(\mathbf{x}, t) = \vec{X}_i(\Phi^{\vec{X}_i}(\mathbf{x}, t)),$$
$$\Phi^{\vec{X}_i}(\mathbf{x}, 0) = \mathbf{x}.$$

Then the time-1 flow $\Phi^{\vec{X}_i}(\mathbf{x}, 1) : \Omega \to \Omega$ is a diffeomorphism.

Now let $\vec{n} := \tilde{\delta}(\phi) \, \tilde{\chi}_A \nabla \phi$, for regularized versions $\tilde{\delta}, \tilde{\chi}_A$ of the Dirac-δ function, and χ_A. We see that $\vec{n}$ coincides with the unit-normal vector field on C. Let η_{ep} be a smooth function on Ω such that $\eta_{ep} = 0$ at the endpoints of the open curves $\Gamma_k \subset C$, $k = 1, 2, ..N$. Consider the vector fields $\vec{Y}_i$ that do not *flow across* C,

$$\vec{Y}_i = P_C X_i := \eta_{ep} \left(\vec{X}_i - (\vec{X}_i \cdot \vec{n}) \vec{n}_i \right).$$

We notice the following properties for the time-1 flow, $\Phi^{\vec{Y}_i}(., 1)$,

- $\Phi^{\vec{Y}_i}(., 1) : \Omega \to \Omega$ is a diffeomorphism since $\vec{Y}_i$ is C^1.
- Also $\vec{Y}_i|_C$ is a C^1 vector field *on* C. Thus $\Phi^{\vec{Y}_i}(., 1)|_C$ is a diffeomorphism onto C.

Hence it is natural to set $f_i = \Phi^{\vec{Y}_i}(., 1)$.

D. Energy

We formulate the energy (1) over the space of C^1 smooth vector fields on Ω, $\vec{X}_i = a_i\frac{\partial}{\partial x} + b_i\frac{\partial}{\partial y}$,
$J[a_i, b_i] =$

$$\int_\Omega |\nabla \tilde{f}_1|^2 \ + |\nabla \tilde{f}_2|^2 \ dx + \lambda \int_\Omega \chi_A\big(\kappa_1(\tilde{f}_1) - \kappa_2(\tilde{f}_2)\big)^2 \ |\nabla H(\phi)| \ |F_x \wedge F_y| \ dx$$
$$+\beta \int_\Omega |D\vec{X}_1|^2 + |D\vec{X}_2|^2 \ dx \tag{2}$$

Here, as before $\tilde{f}_i = f_{0,i}$ o f_i, and $f_i = \Phi^{\vec{Y}_i}(.,1)$, the time-1 flow of the vector field $\vec{Y}_i = P_C X_i$. The last integral in the energy is the smoothness term for the vector fields $\vec{X}_i$.

III. Minimization of the energy

We are going to briefly describe how we derive the Euler-Lagrange equation of the energy functional.

In this work, we formulate the energy functional over the space of C^1 smooth vector fields on Ω, $\vec{X}_i = a_i\frac{\partial}{\partial x} + b_i\frac{\partial}{\partial y}$,
$J[a_i, b_i] =$

$$\int_\Omega |\nabla \tilde{f}_1|^2 \ + |\nabla \tilde{f}_2|^2 \ dx + \lambda \int_\Omega \chi_A\big(\kappa_1(\tilde{f}_1) - \kappa_2(\tilde{f}_2)\big)^2 \ |\nabla H(\phi)| \ |F_x \wedge F_y| \ dx$$
$$+\beta \int_\Omega |D\vec{X}_1|^2 + |D\vec{X}_2|^2 \ dx \tag{3}$$

Here, as before $\tilde{f}_i = f_{0,i}$ o f_i, and $f_i = \Phi^{\vec{Y}_i}(.,1)$, the time-1 flow of the vector field $\vec{Y}_i = P_C X_i$. The last integral in the energy is the smoothness term for the vector fields $\vec{X}_i$. The Euler Lagrange equations of (3) are derived here. Let $D_v^\eta F = \frac{d}{d\epsilon}F(v + \epsilon\eta)$ denote the derivative of a functional F with respect to variable v, and for variation η. Also denote the vector fields $\vec{e}_1 := \frac{\partial}{\partial x}$, $\vec{e}_2 := \frac{\partial}{\partial y}$. It follows that,

$$D_{a_i}J(\eta) = -\int_\Omega \Delta\tilde{f}_i \ Df_{0,i} \ D^\eta_{a_i} f_i \ dx + \lambda\int_\Omega \chi_A(-1)^{i-1}\big(\kappa_1(\tilde{f}_1) - \kappa_2(\tilde{f}_2)\big)$$
$$\nabla\kappa_i Df_{0,i} \ D^\eta_{a_i} f_i |\nabla H(\phi)| \ |F_x \wedge F_y| \ dx + \lambda\int_\Omega \chi_A\big(\kappa_1(\tilde{f}_1) - \kappa_2(\tilde{f}_2)\big)^2$$
$$\frac{1}{|F_x \wedge F_y|}(|F_y|^2 F_x \cdot D^\eta_{a_i} F_x + |F_x|^2 F_x \cdot D^\eta_{a_i} F_y - (F_x \cdot F_y)(F_x \cdot D^\eta_{a_i} F_y + F_y \cdot D^\eta_{a_i} F_x))$$
$$|\nabla H(\phi)| dx - \int_\Omega \Delta a_i \ \eta \ dx \tag{4}$$

Integrating the third term by parts gives,

$$-\lambda\int_\Omega \chi_A \nabla\cdot\big((\kappa_1(\tilde{f}_1) - \kappa_2(\tilde{f}_2))^2 \frac{1}{|F_x \wedge F_y|}[|F_y|^2\partial_x f_i - (F_x \cdot F_y)\partial_y f_i \ ;$$
$$|F_x|^2\partial_y f_i - (F_x \cdot F_y)\partial_x f_i]\big) \ |\nabla H(\phi)| Df_{0,i} \ D^\eta_{a_i} f_i dx \tag{5}$$

In the first two integrals, the term $D^\eta_{a_i} f_i$ is given by the flow equation of $\vec{Y}_i = P_C X_i$,

$$\frac{\partial \Phi^{\vec{Y}_i}}{\partial t}(x,t) = \vec{Y}_i(\Phi^{\vec{Y}_i}(x,t)),$$
$$\Phi^{\vec{Y}_i}(x,0) = x.$$

Now computing the derivative with respect to a_i on both sides, for variation η gives the following differential equation, $P_i := D^{\eta}_{a_i}\Phi^{\vec{Y_i}}$,

$$\frac{\partial}{\partial t}P_i(x,t) = \eta P_C\vec{e}_1\ (\Phi^{\vec{Y_i}}(x,t)) + D\vec{Y_i}(\Phi^{\vec{Y_i}}(x,t))\ P_i(x,t),$$
$$P_i(x,0) = \mathbf{0}. \tag{6}$$

Since $D\vec{Y_i}(\Phi^{\vec{Y_i}}(x,t))$ is continuous with respect to t, we have the existence of an orthogonal fundamental matrix Ψ_i, for the homogeneous system of (6). Then a solution for the above problem can be easily verified to be

$$P_i(x,t) = \Psi_i(x,t)\ \int_0^t \Psi_i^{-1}(x,s)P_C\vec{e}_1\ (\Phi^{\vec{Y_i}}(x,s))\eta(\Phi^{\vec{Y_i}}(x,s))\ \ ds$$

Let $B_i := -\Delta\tilde{f}_i\ Df_{0,i}\ +\ \lambda\chi_A\ \big((-1)^{i-1}\big(\kappa_1(\tilde{f}_1)-\kappa_2(\tilde{f}_2)\big)\nabla\kappa_i - \nabla\cdot C_i\big)\ Df_{0,i}\ |\nabla H(\phi)|$,
where $C_i = (\kappa_1(\tilde{f}_1)-\kappa_2(\tilde{f}_2))^2\ \frac{1}{|F_x\wedge F_y|}[|F_y|^2\partial_x f_i - (F_x\cdot F_y)\partial_y f_i\ ; |F_x|^2\partial_y f_i - (F_x\cdot F_y)\partial_x f_i]$. Substituting $D^{\eta}_{X_i} f_i = P_i(.,1)$ in (4), we have

$$D_{a_i}J(\eta) = \int_\Omega B_i(x)\ \Psi_i(x,1)\ \int_0^1 \Psi_i^{-1}(x,s)\ P_C\vec{e}_1\ (\Phi^{\vec{Y_i}}(x,s))\ \eta(\Phi^{\vec{Y_i}}(x,s))\ \ ds\ dx$$
$$-\int_\Omega \Delta a_i\ \eta\ dx$$

For a fixed s, $\Phi^{\vec{Y_i}}(y,s):\Omega\rightarrow\Omega$, is a diffeomorphism; denote the inverse map by $\phi^{\vec{Y_i}}_s$, and its Jacobian by $|D\phi^{\vec{Y_i}}_s|$. Change of variables $(x,s)\rightarrow(y=\Phi^{\vec{Y_i}}(x,s),s)$ in the first term gives

$$\int_\Omega B_i(\phi^{\vec{Y_i}}_s(y))\ \int_0^1 \Psi_i(\phi^{\vec{Y_i}}_s(y),1)\ \Psi_i^{-1}(\phi^{\vec{Y_i}}_s(y),s)\ P_C\vec{e}_1\ (y)\ |D\phi^{\vec{Y_i}}_s(y)|\ \eta(y)\ \ ds\ dy$$

Thus the Euler Lagrange equations are

$$\frac{da_i}{dt} = \int_0^1 B_i(\phi^{\vec{Y_i}}_s)\ \Psi_i(\phi^{\vec{Y_i}}_s,1)\ \Psi_i^{-1}(\phi^{\vec{Y_i}}_s,s)\ P_C\vec{e}_1\ \ |D\phi^{\vec{Y_i}}_s|\ ds - \beta\Delta a_i$$
$$\frac{db_i}{dt} = \int_0^1 B_i(\phi^{\vec{Y_i}}_s)\ \Psi_i(\phi^{\vec{Y_i}}_s,1)\ \Psi_i^{-1}(\phi^{\vec{Y_i}}_s,s)\ P_C\vec{e}_2\ \ |D\phi^{\vec{Y_i}}_s|\ ds - \beta\Delta b_i,$$

where: $B_i := -\Delta\tilde{f}_i\ Df_{0,i}\ +\ \lambda\chi_A\ \big((-1)^{i-1}\big(\kappa_1(\tilde{f}_1)-\kappa_2(\tilde{f}_2)\big)\nabla\kappa_i - \nabla\cdot C_i\big)\ Df_{0,i}\ |\nabla H(\phi)|$; Ψ_i is the orthogonal fundamental matrix for the homogeneous system of

$$\frac{\partial}{\partial t}P_i(x,t) = \eta P_C\vec{e}_1\ (\Phi^{\vec{Y_i}}(x,t)) + D\vec{Y_i}(\Phi^{\vec{Y_i}}(x,t))\ P_i(x,t),\ \ P_i(x,0) = \mathbf{0}.$$

IV. Experimental Results

We have tested our algorithm on synthetic surfaces. Figure 4 shows the matching result of the synthetic surfaces with two sharp corners. Figure 4(A) shows a synthetic surface. It is mapped to another synthetic surface through parameterizations without the shape based correspondence between landmark curves, as shown in (B). Note that the correspondence between the landmark curves does not follow the shape information ([See the yellow dot]). (C) shows the result of matching using our proposed algorithm. Note that the correspondence between the landmark curves follows the shape information (corners to corners [See the yellow dot]). Figure 5 shows the matching result of the synthetic surfaces with three sharp corners. (A) shows one synthetic surface with three sharp corners. Again, it is mapped to another synthetic surface through parameterizations without the shape based correspondence between landmark curves, as shown in (B). The correspondence between the landmark curves does not follow the shape information. (C) shows

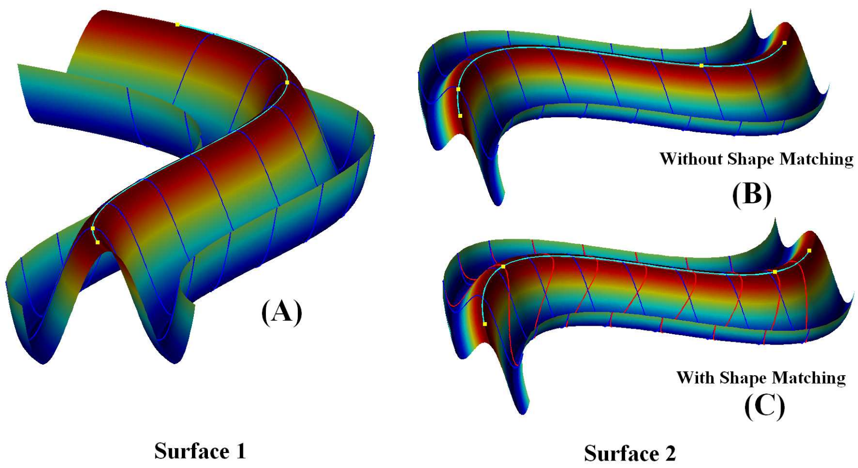

Fig. 4. The figure shows matching result of the synthetic data with two sharp corners.

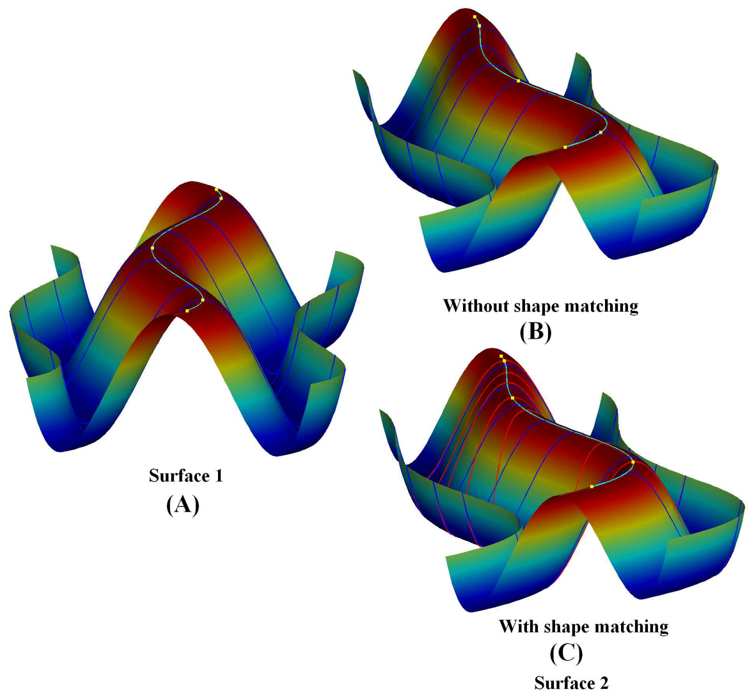

Fig. 5. The figure shows matching result of the synthetic data with three sharp corners.

the result of matching using our proposed algorithm. The correspondence between the landmark curves follows the shape information.

We have also tested our algorithm on cortical hemispheric surfaces extracted from brain MRI scans, acquired from normal subjects at 1.5 T (on a GE Signa scanner). Experimental results show that our algorithm can effectively compute cortical surface parameterizations that align the landmark features in a way that also enforces shape correspondence, while preserving the conformality of the surface-to-surface mapping to the greatest extent possible. The computed map is guaranteed to be a diffeomorphism because the map is formulated as the integral flow of a smooth vector field.

Figure 6 shows two different cortical surfaces with sulcal landmarks labeled on them. We seek parameterizations of these surfaces that align the landmark features consistently while optimally preserving conformality. A diffeomorphism between the two surfaces is then obtained by computing the composition

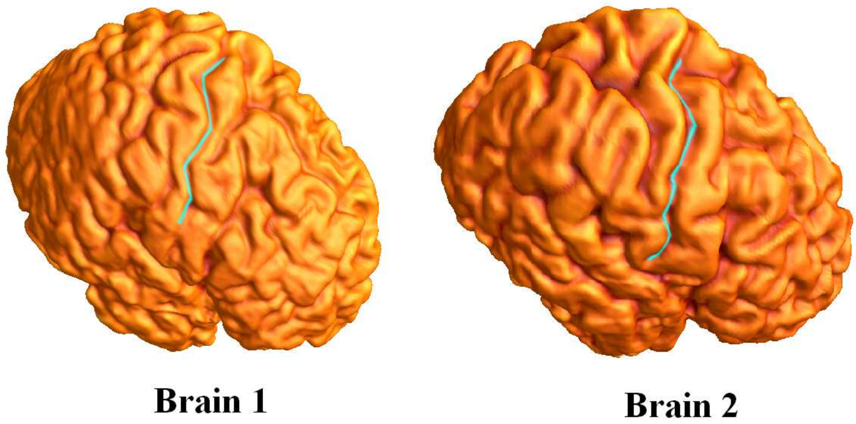

Fig. 6. The figure shows two different cortical surfaces with sulcal landmarks.

of the two parameterizations. Figure 7 shows the result of matching the cortical surfaces with one landmark labeled (for purposes of illustration) on each brain. Figure 7(A) shows the cortical surface of Brain 1. It is mapped to the cortical surface of Brain 2 under the conformal parameterization as shown in Figure 7(B). Note that the sulcal landmark on Brain 1 is only mapped approximately to the sulcal region on Brain 2. It is not mapped exactly to the corresponding sulcal landmark on Brain 2. Figure 7(C) shows the matching result under the parameterization we propose in this paper. Note that the corresponding landmarks are mapped exactly. Also, the correspondence between the landmark curves follows the shape information. It maps the secondary features of one landmark curve to the secondary features of the other landmark curve (See the black dots). Figure 7(D) and (E) show the standard 2D parameter domain of Brain 1 and Brain 2 respectively. The landmark curve is mapped to same horizontal line and the shape feature are mapped to the same positions (see the black dots). This is advantageous as the surface average of many subjects would retain features that consistently occur on sulci, while uniform speed parameterizations may cause these features to cancel out (please see Figure 1 for illustration). Figure 8 gives an illustration of the matching results for cortical surfaces with several sulcal landmarks labeled on them. Figure 8(A) shows the brain surface 1 with several landmarks labeled. It is mapped to brain surface 2 under the conformal parameterization as shown in Figure 8(B). Again, the sulcal landmarks on Brain 1 are only mapped approximately to the sulcal regions on Brain 2. Figure 8(C) shows the matching result under the parameterization we proposed. The corresponding landmarks are mapped exactly. Also, the correspondence between the landmark curves follows the shape information (corners to corners [See the black dot]). To examine the conformality of the parameterization, we show in Figure 9($\mathbf{Left}$) the histogram of $g_{12} = g_{21}$ of the Riemannian metric under the parameterization computed with our proposed algorithm. Observe that $g_{12} = g_{21}$ are very close to zero at most vertices. This means that the Riemannian metric is a diagonal matrix, thus the parameterization computed is very close to conformal. It also shows that conformal map being intrinsic to global surface geometry, is not significantly affected by small changes in the local geometry induced by the shape term. Figure 9($\mathbf{Right}$) shows that the shape energy is decreasing with iterations, implying an improving shape based correspondence between the landmark curves.

V. Conclusion and future work

In this paper, we developed an algorithm to find parametrizations of the cortical surfaces that are close to conformal and also give a *shape based* correspondence between embedded landmark curves. We propose a variational approach by minimizing an energy that measures the harmonic energy of the parameterization *maps*, and the shape dissimilarity between mapped points on the landmark curves. The parameterizations computed are guaranteed to give a *shape-based* diffeomorphism between the landmark curves. Experimental results show that our algorithm can effectively compute parameterizations

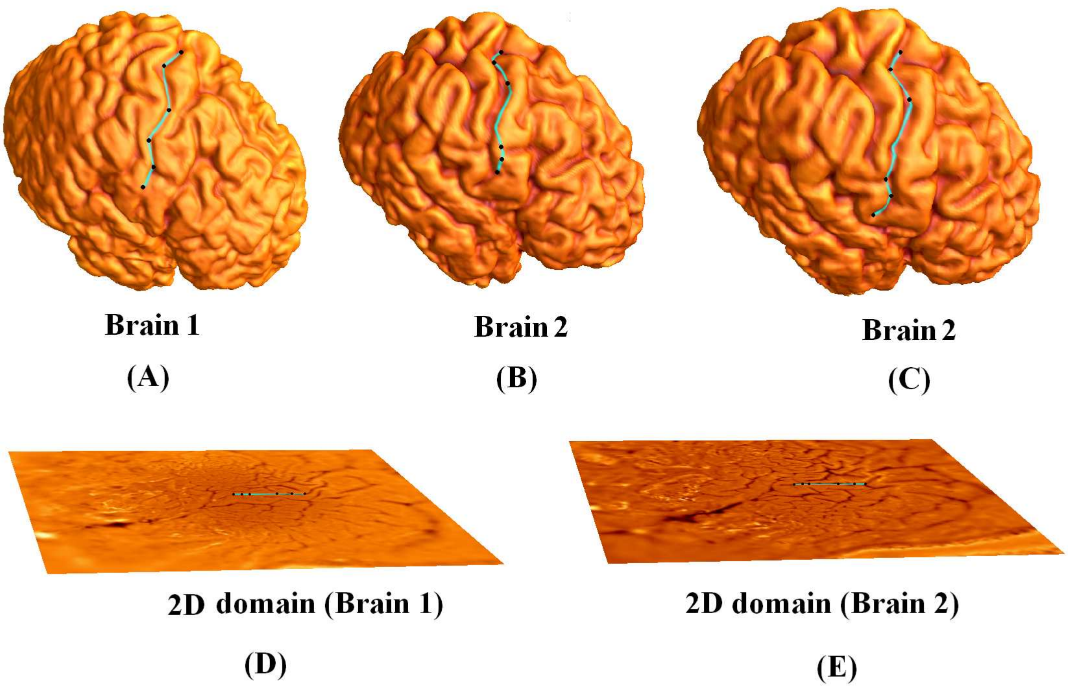

Fig. 7. This figure shows the result of matching the cortical surfaces with one landmark labeled. (A) shows the surface of Brain 1. It is mapped to Brain 2 under conformal parameterization, as shown in (B). (C) shows the result of matching using our proposed algorithm. (D) and (E) show the standard 2D parameter domains for Brain 1 and Brain 2 respectively.

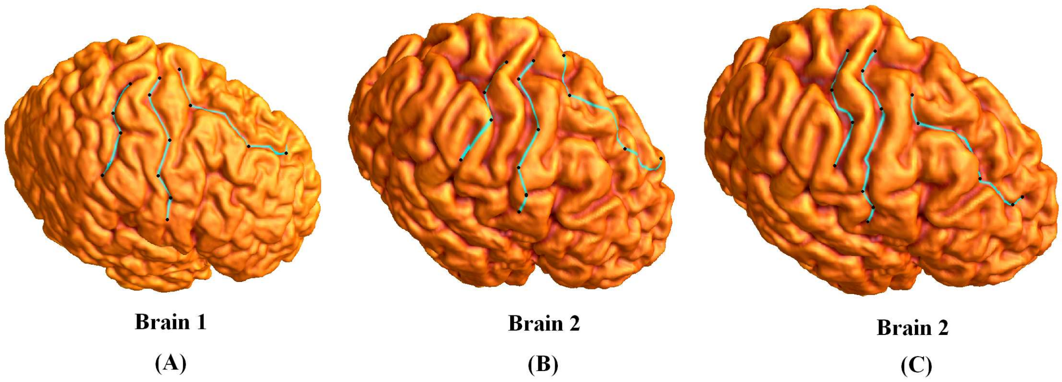

Fig. 8. Illustration of the result of matching the cortical surfaces with several sulcal landmarks. (A) shows the brain surface 1. It is mapped to brain surface 2 under the conformal parameterization as shown in (B). (C) shows the result of matching under our proposed parameterization.

of cortical surfaces that align landmark features consistently with shape correspondence, while preserving the conformality as much as possible. As future work, we plan to apply this algorithm to cortical models from healthy and diseased subjects to build population shape averages. The enforcement of higher-order shape correspondences may allow subtle but systematic differences in cortical patterning to be detected, for instance in neurodevelopmental disorders such as Williams syndrome, where the scope of cortical folding anomalies is of great interest but currently unknown. Another area of interest is to work on better numerical schemes to improve computational efficiency and accuracy.

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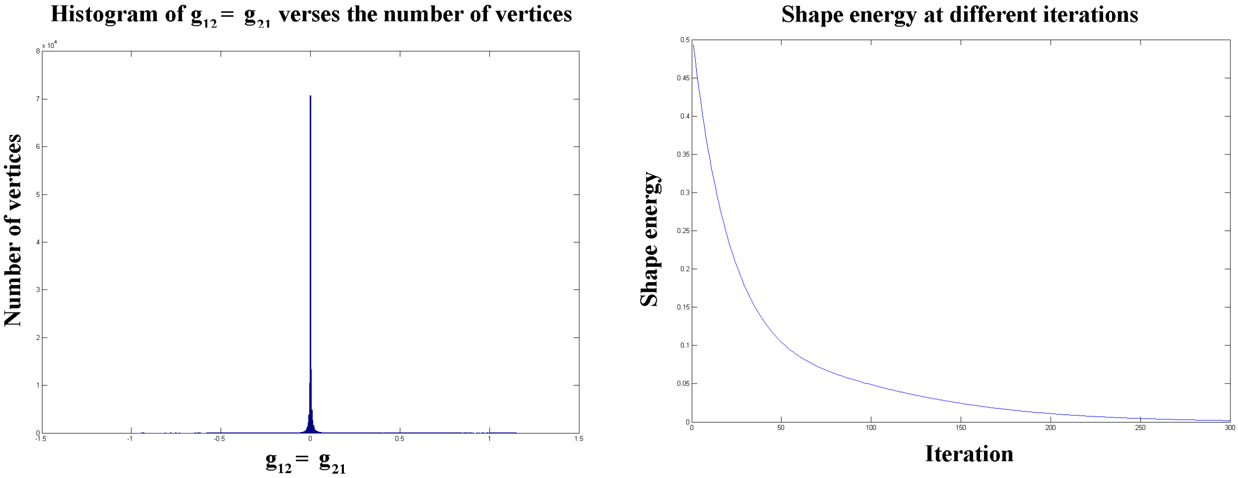

Fig. 9. The left shows the histogram of $g_{12} = g_{21}$ of the brain surface under the parameterization computed with our algorithm. The right shows the shape energy at different iterations.

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