MULTI-PHASE AND MULTI-CHANNEL REGION SEGMENTATION AND APPLICATION IN BRAIN MRI

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Abstract In this work, we consider the segmentation of images as the minimization of an energy involving a regionbased data fidelity term and a regularization term, as the Chan and Vese model. We focus on two aspects of the problem: optimization and fusion of the information provided by several sources. A first contribution consists in correcting the multi-phase level-set based approach proposed by Chan and Vese from the so-called "hidden phase problem". We later compare it to the Iterated Conditional Mode (ICM) and the Simulated Annealing (SA) algorithms. Our second contribution is the extension of the multi-phase model to the multi-channel case, which we study from a semantic point of view, allowing only relevant combinations of the different channels' data. Experiments on real MRI brain data, for normal patients and for pathological cases, show that ICM is faster and more accurate than the level sets and that multiprotocol MR images can be used in our multi-phase and multi-channel model to segment both normal and pathological structures.

Keywords Segmentation · Optimization · Mumford-Shah functional · Multi-channel fusion · Multi-phase level sets · Brain imaging

1 Introduction

Image segmentation is an early vision problem that has been intensively studied. A standard approach consists in minimizing an energy that is a weighted combination of a data

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fidelity term which measures the closeness of the segmentation to the observed data and a regularization term which embeds prior knowledge about the solution. This approach was originally proposed in (Geman and Geman, 1984). In this seminal work, the authors formalize the problem in a discrete optimization framework and more precisely as a Markov Random Field (MRF). They use the famous Ising (binary labelling) and Potts (multi-label) models to penalize discontinuities of labels in the solution. Another approach consists in using Minimum Description Length (MDL) as proposed in (Leclerc, 1989) to model the prior. Note that the latter is not necessarily Markovian and is thus different from the approach of Geman and Geman. In (Mumford and Shah, 1989), Mumford and Shah introduced their acclaimed model, which can be seen as a continuous version of the work of Geman and Geman. Contrary to the discrete formalism, this continuous approach allows a better understanding of the model as well as to study more easily the nature of the minimizers (although the analysis and computation of minimizers for the generic model remains quite challenging), see (Aubert and Kornprobst, 2006) for instance and (David, 2005) for a full theoretical study. In (Zhu and Yuille, 1996), the authors proposed a framework that combines these approaches in a unifying segmentation framework. Research works exploiting these approaches for image segmentation have mainly focused on the regularization part, and the introduction of prior models (e.g. shape models), while using data-specific fidelity terms, which depend on the application.

These models generally lead to difficult optimization problems either with discrete or continuous approaches. A generic discrete optimization approach for minimizing discrete energies, that has been originally proposed in (Geman and Geman, 1984), consists in defining a sampler that explores the solution space and embedding it into a Simulated Annealing (SA) process which is characterized by a dynamic tempera2

ture parameter. Several sampling strategies can be adopted, including the well known Gibbs and the Metropolis samplers. This approach is known to converge towards a global minimizer of the energy, provided the decrease of temperature is sufficiently slow. We refer the reader to (Winkler, 2006) for a full presentation and studies of these discrete optimization approaches. Although SA approaches have good theoretical properties they are generally computationally consuming. To alleviate this constraint, Besag proposed in (Besag, 1986) the Iterated Conditional Mode (ICM) approach that mainly consists in setting the temperature parameter to zero in the SA approach. This strategy does not guarantee anymore the convergence towards a global minimizer but drastically improves the computational burden. For the case of binary segmentation, related to the Ising model, Markovian energies can be globally optimized using a standard graph-based approach as described in (Boykov et al, 2001; Picard and Ratlif, 1975) for instance. We also note the nice approximation property of the graph-based approach of (Boykov stand what constitutes an object, since a given structure may et al, 2001) for the Potts model.

In (Zhu and Yuille, 1996), the authors considered a hybrid scheme in which the energy is iteratively minimized, by introducing random seed points, moving the boundaries of competing regions and then by performing a greedy reduction of the number of regions.

From a continuous point of view, the most popular optimization approach relies on the use of the level-set framework (Osher and Sethian, 1988) to perform interface evolution. It is well known that the main advantage of this framework over the traditional snake approach (Kass et al, 1988) is that it naturally copes with the change of topology. This technique has been used in the Active Contour Without Edges (ACWE) model of (Chan and Vese, 2001) for the case of binary segmentation, and in (Vese and Chan, 2002) for the multi-label (i.e. multi-phase) case. Note that the two latter can be seen as continuous versions of the discrete Ising and Potts models, respectively. In this paper we consider the multi-phase segmentation and a continuous or discrete point of view is adopted depending on the considered optimization technique.

Regarding the *data fidelity term*, most studies consider scalar images and focus on the regularization model. However, the evolution of technology now provides researchers with many different imaging modalities to study a given scene or object. This is especially true of living organisms, for which different types of waves and interactions are used to obtain images of various physiological properties, through MRI, X-ray scanner or scintigraphy (PET and SPECT) for example. For a given modality, various acquisition protocols provide the physician with a large and diverse set of information regarding organs or anatomical structures. Examples include the use of different T1 and T2 weightings in MRI, contrast agents, or many different radiotracers in scintigra-



Fig. 1 T1-weighted (a) and T2-weighted (b) images of a brain with tumor and edema. While the pathological area, defined as the tumor and the edema, appears as one single region, very contrasted from the other tissues in the T2-weighted image (in white), the same area decomposes in the T1-weighted image into the tumor (dark gray) and the surrounding edema (lighter gray, hardly distinguishable from gray matter).

phy. Multi-modality imaging provides more information but also leads to a dramatic change as to how we are to underappear with varying shape or contrast in the different images.

In the more general context of multi-channel imaging, image interpretation needs to combine different types of information from a given scene. In this context, the segmentation problem can become non-univocal, given the different and somehow contradictory - information that is provided by the different modalities corresponding to different channels. Hence, in MRI pathological brain imaging for example, a tumor will appear as an homogeneous region in T1-weighted images, distinguishable from the surrounding edema, while in T2-weighted images, the tumor and edema constitute a single region, very contrasted from the rest of the brain (see Figure 1). One can therefore decide to focus on the segmentation of the tumor solely and choose to take the edema away from the segmented object. However, one can also decide to group both the tumor and edema in a single region that represents the pathological area. These two strategies formally correspond to two possible combinations of visual information given by the two channels regarding the tumoral region, namely intersection and union.

Yet, the motivation for using multi-channel or multi-modality images is that one obtains complementary information concerning possibly many different objects of the image. In the example of pathological brain MRI that we are primarily interested in, a possible gain of using multi-protocols over single-protocol MRI is to extract the information provided by T1-weighted images to separate white matter (WM) from gray matter (GM), while demarcating the edema from GM by making use of the contrast available in the T2-weighted images.

Working in the framework of energy minimization for multi-phase segmentation, the choice of the data fidelity term should be based on its simplicity, generality and its applicability to multiple objects. This last requirement eliminates gradient-based data terms, which use derivatives and therefore ignore actual values of the image inside different connected components defined by the set of discontinuities. Hence, gradient information can be used to find borders of several objects but cannot assign different labels on them. For this reason, we chose a region-based energy functional, which characterizes homogeneous regions through their average intensity values, combined with a regularization term. For a given image u_0 defined in Ω , the model consists in finding P regions R_j such that $\bigcup_{j=1}^{P} R_j = \Omega$, with associated contours ∂R_j and means c^j , minimizing the energy:

$$E = \sum_{j=1}^{P} \int_{R_j} (u_0 - c^j)^2 \mathrm{d}x + \nu \cdot \mathrm{Length}(\partial R_j), \qquad (1)$$

which is the functional associated to the "minimal partition problem" proposed in (Mumford and Shah, 1989), and is a restriction of the general Mumford and Shah functional to the piecewise constant case.

Our goal in this paper will therefore be to develop a robust multi-region (called "multi-phase" in the level set framework) segmentation method based on the minimization of this energy and to later study the possibilities to extend it to the multi-channel case, in order to find distinct objects in the whole scene, according to the information given by at least two registered images of this scene.

A first contribution of this work is to identify and solve an issue with the multi-phase ACWE (the so-called *hidden phase problem*), related to a specific kind of local minimum in the level sets framework. We also provide a comparison of the corrected multi-phase ACWE with the ICM algorithm (discrete framework), and demonstrate that the latter method can achieve similar or even better energy minimization results, but a lot faster, and with more flexibility on the number of phases. Finally, we study the possibilities to extend any of these two multi-phase segmentation frameworks to the multi-channel case and propose a fusion rule that is associated with a clear semantics, namely an intersection rule applied in all the regions.

The remainder of this paper is as follows: In Section 2, we review works that are related to either the optimization problem, in both continuous and discrete approaches, or multi-channel segmentation. In Section 3, we describe the two aforementioned methods to find a minimum to equation (1): the multi-phase ACWE, free of the hidden phase problem and the ICM algorithm and show a comparison between the two, in terms of qualitative and quantitative results on brain MRI and computation times. In Section 4, we propose an extension of multi-phase segmentation framework to the multi-channel case. This extension is used in Section 5 to perform multi-phase and multi-channel segmentation

on pathological brain MRIs with T1 and T2 weighted protocols. Finally, we draw some conclusions in Section 6.

2 Related works

In order to perform the minimization of the energy defined in Equation (1), we propose to compare two classes of methods, which have both been largely described in the literature, and which are based respectively on a continuous and discrete formulation of the energy (1).

- 1. In its continuous formulation, Equation (1) can be minimized by a variational method in the level-set formalism (Osher and Sethian, 1988), which guarantees topological flexibility of the contours. Only after Euler-Lagrange evolution equations have been derived do we discretize the system. Here, we follow the work of Chan and Vese, with particular regard to the multi-phase extension (Vese and Chan, 2002) of their Active Contours Without Edges (ACWE) method (Chan and Vese, 2001). We point out here that for the piece-wise constant Mumford-Shah energy functional, corresponding to the ACWE framework, a minimizer exists for a finite number of regions. Regarding level-set based multiphase segmentation frameworks, related to the ACWE energy functional, three families of approaches have been studied: (1) multiple independent level set functions (i.e. N functions for Nphases) as in (Samson et al, 2000), (Zhao et al, 1995) (2) joint level set functions (i.e. N functions for 2^N phases), as in (Vese and Chan, 2002), (Cremers et al, 2006) (3) a single multilayers level set function (i.e. 1 level set function for N phases) as in (Chung and Vese, 2005). The multilayer approach benefits from a smaller computational cost than the original multi-phase ACWE but forces a nested structure of the segmented regions. Since we wish to remain as general as possible in this work, we did not employ this method, but, as stated by the authors, a nested structure can be useful in many applications. We follow the formulation of Chan and Vese, defining 2^N phases as the intersection of the positive and negative parts of N level set functions which permits one to automatically cope with the problem of vacuum and overlap, since one obtains a partition of the image.
- 2. In its *discrete formulation*, the energy (1) can be minimized by a clustering algorithm similar to the k-means, with an additional regularization process as proposed by the *Iterated Conditional Mode* (ICM) algorithm (Besag, 1986). Since no deterministic algorithm is known to provide a global minimum for the multi-phase energy (1) in polynomial time, we also ran a Metropolis sampler (Winkler, 2006) into a simulated annealing algorithm (Kirkpatrick et al, 1983; Kirkpatrick, 1984), which is

known to provide an accurate order of magnitude of the energy level of the global minimum.

A recent study has compared continuous versus discrete optimization methods for multiphase segmentation in (Szeliski et al, 2008). Our work adds up to this study, comparing the performance of several algorithms in terms of their capacity to decrease the energy associated with a particular multiphase model.

From an application point of view, we report several works which use an energy based approach for brain and/or brain tumor segmentation. Parametric deformable models or snakes, proposed in (Kass et al, 1988) have been widely used, but are limited to the detection of a single object. The segmented object can have several connected components, although the method only handles such cases with difficulty, but it is not possible to decompose an image into more than two parts (object and background) from the information given by gradients only. Hence, snakes have been used for the segmentation of brain tumors in (Luo et al, 2003) and (Jiang et al, 2004) among others. Level set based geometric deformable models avoid the burden of parameterization, therefore handling topological changes automatically. They are used in (Droske et al, 2001), (Cates et al, 2004), (Lefohn et al, 2003) and (Xie et al, 2005) for brain tumor segmentation.

Among the few recent works which have explored multiphase segmentation of brain MRI data, based on the minimization of the Mumford-Shah energy, we report the work of (Jeon et al, 2005), in which multi-phase segmentation is achieved by successively applying the 2-phase ACWE in subregions of the original image. The second class of methods for energy minimization, based on Markov Random Fields, has been used in the work of (Shen et al, 2003), in pathological brain imaging, where pixels are classified into ten clusters which are later merged into four classes. These classes correspond to CSF, GM, WM and the tumor. (Held et al, 1997) have taken a similar approach for use in brain MR images. Five classes, corresponding to the background, WM, GM, CSF and scalp-bone and other non-brain tissue are segmented according to a model that also takes signal inhomogeneities into account. Simulated annealing and ICM algorithms were used in this work and the former method shows better results, but with significantly longer computation times. Multi-phase brain tissue segmentation, was evaluated in several recent works, including (Cheng et al, 2005), which combines both region and gradient information in ACWE model, (Angelini et al, 2007b) where the ACWE performance was compared to other segmentation methods and in (Angelini et al, 2006) where different homogeneity measures were evaluated for the brain segmentation task.

Regarding vectorial or multi-channel extensions, two recent methods have been proposed for the binary (2-phase) case. The first model (ACWE for vector valued images, (Chan et al, 2001)) simply proposes to average the heterogeneity measure over the channels, but this model presents a risk of oscillation between several equally good solutions and highly depends on the initialization. The second model (the logic framework, (Sandberg and Chan, 2005)) fixes this problem by defining complementary rules of fusion inside and outside the contour, leading to a clear and flexible method that permits one to obtain the union or intersection of several observations of a same object in different channels. It remains to see how this work can be extended to the multiphase case, where the exterior of a contour is not to be considered as a single region. We note, however, that the approach of Jeon et al. permits one to directly extend this multichannel segmentation method to the multi-phase case, by making use of their binary hierarchical scheme. Other attempts to integrate several modalities in the segmentation process can be found in (Wasserman and Acharya, 1995). This paper adds a region term (the so-called local region influence (LRI)) to the external force acting upon a deformable model (snake). Fusion is achieved by summing the LRIs over the channels, which is also the approach taken in (Chan et al, 2001). Although an improvement is demonstrated over single-channel segmentation, this work suffers from the same risk of oscillation as the vector valued model of Chan et al. and the snake approach cannot be easily extended to the multi-phase case. A multi-channel approach has also been found to enhance the discrimination between different tissue classes in the work of (Rajapakse et al, 1996), based on a statistical approach. In the work of (Wu et al, 2006), the knearest neighbor classification algorithm is extended to handle three-channel data, combining proton density-, T2- and contrast enhanced T1-weighted brain images. The method was found to increase intra-class correlation coefficients, as well as identification and segmentation accuracy of three subtypes of lesions over two-channel segmentation, where the intensity vector from the third channel (T1) was removed. Here, the fusion between the channels is performed by considering the Euclidean distance to a class in respectively three- and two-dimensional spaces. This corresponds to an average of the information carried by the channels, meaning that a point can be classified in a given phase even if its gray level intensity differs greatly in one channel, so long as the difference is small in the other channel(s). For a review of other works related to multi-protocol MRI brain tumor segmentation, see also (Angelini et al, 2007a; Khotanlou, 2008).

3 Optimization for the multi-phase case

In this work, we consider the problem of minimizing the energy (1) for a fixed number of regions P. We detail now two methods: the multi-phase ACWE and the ICM algorithm.

3.1 Level Set based formulation: the multi-phase ACWE

The multi-phase ACWE is an extension of the 2-phase ACWE, in which a simplified form of Equation (1) is minimized. The 2-phase ACWE intends to approximate a signal with another one that can only take two values c^1 and c^2 while enforcing the smoothness of the boundaries. Formally, let Ω be a bounded subset of \mathbb{R}^n , n = 2, 3, with $\partial\Omega$ its boundary and $\overline{\Omega}$ its closure, then, for a given image $u_0 : \overline{\Omega} \to \mathbb{R}$ and a curve or surface C in Ω , the energy to minimize for the evolving curve to fit an object in the image writes (for n = 2) as follows:

$$F(c^{1}, c^{2}, C) = \nu \cdot \text{Length}(C) + \mu \cdot \text{Area}(inside(C)) + \int_{inside(C)} |u_{0}(x) - c^{1}|^{2} dx + \int_{outside(C)} |u_{0}(x) - c^{2}|^{2} dx ,$$

$$(2)$$

where c^1 and c^2 are two real numbers. The level set formulation of this minimization problem has now become standard and leads to a gradient descent algorithm parameterized by an artificial time $t \ge 0$, derived from the Euler-Lagrange equations. At each step, c^1 and c^2 are computed as the average value respectively inside and outside the contour C, which is now represented by the zero level of a Lipschitz scalar function Φ defined on Ω . The evolution of the contour is then implicitly defined by the evolution of Φ , according to

$$\frac{\partial \Phi}{\partial t} = \delta_{\epsilon} \left\{ \nu \operatorname{div} \left(\frac{\nabla \Phi}{|\nabla \Phi|} \right) - \mu - (u_0 - c^1)^2 + (u_0 - c^2)^2 \right\}$$
(3)

where δ_{ϵ} is a regularized version of the Dirac function.

The multi-phase ACWE is a straightforward generalization of the 2-phase model of (Vese and Chan, 2002). The basic idea is to evolve N level set functions instead of one, which defines up to 2^N phases, when considering the possible intersections of all positive and negative regions of each level set function. This way to proceed shows several good properties. The first one is to be computationally efficient since only three level set functions are needed to define up to eight phases for example. The second one is that exactly the same formalism as for the 2-phase model is used, with hardly any further sophistication. Finally, defining phases by the intersection of the positive and negative regions associated with the level set functions permits one to keep the valuable property of obtaining a partition of the image, with no vacuum or overlap.

The drawback of this formulation is that phases are defined implicitly, from the intersections of the level set functions being manipulated. Therefore, a global change on one of these functions may affect all phases. Reciprocally, changing one phase in a certain way may require that we evolve several level set functions.

One first difficulty that arises from the fact that phases are defined implicitly in the multi-phase model of Chan and Vese is that the length term is hard to compute. This problem has been pointed out by the authors in (Chan and Vese, 2001), who proposed to change the length of the contour of the phases by the sum of the length of the zero-level sets, and stated that this does not affect the quality of the results.

The formulation of the multi-phase energy, for four phases, is defined with two level set functions $\Phi = (\phi_1, \phi_2)$ (we will later propose a general formulation for an arbitrary number of level set functions). The average values $c = c_{i,j}$ are defined for i, j = 0, 1, according to the sign of resp. ϕ_1 and ϕ_2 . Then, the energy writes

$$F(c, \Phi) = \iint_{\Omega} (u_0(x) - c^{1,1})^2 H(\phi_1) H(\phi_2) dx$$

+
$$\iint_{\Omega} (u_0(x) - c^{1,0})^2 H(\phi_1) (1 - H(\phi_2)) dx$$

+
$$\iint_{\Omega} (u_0(x) - c^{0,1})^2 (1 - H(\phi_1)) H(\phi_2) dx \qquad (4)$$

+
$$\iint_{\Omega} (u_0(x) - c^{0,0})^2 (1 - H(\phi_1)) (1 - H(\phi_2)) dx$$

+
$$\nu \iint_{\Omega} |\nabla H(\phi_1)| + |\nabla H(\phi_2)| dx.$$

Iterative minimization of this energy is performed by alternating between the computation of the vector c, whose components are the average values on the different phases, and the evolution of the two level set functions ϕ_1 and ϕ_2 via their associated Euler-Lagrange equations:

$$\begin{split} \frac{\partial \phi_1}{\partial t} = & \delta_{\epsilon}(\phi_1) \Big\{ \nu \operatorname{div} \left(\frac{\nabla \phi_1}{|\nabla \phi_1|} \right) \\ & - \left[((u_0 - c^{1,1})^2 - (u_0 - c^{0,1})^2) H(\phi_2) \right. \\ & + ((u_0 - c^{1,0})^2 - (u_0 - c^{0,0})^2) (1 - H(\phi_2)) \right] \Big\}, \\ \frac{\partial \phi_2}{\partial t} = & \delta_{\epsilon}(\phi_2) \Big\{ \nu \operatorname{div} \left(\frac{\nabla \phi_2}{|\nabla \phi_2|} \right) \\ & - \left[((u_0 - c^{1,1})^2 - (u_0 - c^{1,0})^2) H(\phi_1) \right. \\ & + ((u_0 - c^{0,1})^2 - (u_0 - c^{0,0})^2) (1 - H(\phi_1)) \right] \Big\}. \end{split}$$
(5)

Formulation of the energy functional as well as evolution equations become tedious to write in this form when more than two level set functions are used. In order to generalize those formulas to the 2^N -phase case, we need to introduce new notations. This will also be useful to implement a code in which N can be chosen arbitrarily. The idea of these notations is to establish a correspondence between the signs of the level set functions, represented by 1 (positive) or 0 (negative) and binary representation of numbers.

Generalization to 2^N phases In the multi-phase model with N level set functions, each phase $j = 0, \dots, 2^N - 1$ is defined by the intersection between N sets, each of which corresponding either to the positive or negative part of a level set function. Defining a phase therefore amounts knowing what the sign of each level set function is. For that matter, as well as to shorten the notations, we need to introduce three definitions. The first one establishes the correspondence between the index of a phase and the sign of the N level set functions. For instance, if N = 3 and the considered phase is j = 5, considering its binary representation j = 101 gives a clear rule to define the signs of the 3 level set functions, namely $\phi_1 > 0, \phi_2 < 0$ and $\phi_3 > 0$. Then, the second definition is aimed at combining the right characteristic functions χ_l^j for phase j, according to the sign of each level set function l. Finally, the third definition does the same job as the second one, but for the sake of the dynamical scheme, where a level set function ϕ_l evolves according to the difference between heterogeneity measures inside and outside $\{\phi_l > 0\}$, in all the phases defined by the signs of the other level set functions $h_1 = 1, \ldots, l-1$ and $h_2 = l+1, \ldots, N$. We now formally state the three definitions.

Definition 1 For a positive integer N, let the integer $j \in [0, 2^N - 1]$. For each $1 \le m \le N$, let $B^j(m)$ the m^{th} bit (binary digit) of j, so that the vector B^j can be identified to a binary representation of j. Reciprocally, given a binary vector B of length N, we write \hat{B} , the number $B(N) \times 2^{(N-1)} + \ldots + B^j(1) \times 2^0$.

Definition 2 Given a positive integer N, for all $0 \le j \le 2^N - 1$, we define the vectors B^j in $\{0,1\}^N$ according to Definition 1. For N functions $\Phi_l : \Omega \mapsto \mathbb{R}$, $1 \le l \le N$, let

$$\chi_{l}^{j} = \begin{cases} H(\phi_{l}) & \text{if } B^{j}(l) = 1\\ 1 - H(\phi_{l}) & \text{if } B^{j}(l) = 0 \end{cases}$$
(6)

Then we define

$$\chi_j = \prod_{1 \le l \le N} \chi_l^j \tag{7}$$

Definition 3 Given a positive integer N, for all $1 \le l \le N$, let $h_1 \in [0, 2^{l-1} - 1]$ and $h_2 \in [0, 2^{N-l} - 1]$ and, respectively, the vectors B^{h_1} and B^{h_2} of length l - 1 and N - las in Definition 1. For N functions $\phi_l : \Omega \mapsto \mathbb{R}$ and for $1 \le p \le N$, if p < l, let

$$\chi_p = \begin{cases} H(\phi_p) & \text{if } B^{h_1}(p) = 1\\ 1 - H(\phi_p) & \text{if } B^{h_1}(p) = 0 \end{cases}$$
(8)

and if p > l, let

$$\chi_p = \begin{cases} H(\phi_p) & \text{if } B^{h_2}(p) = 1\\ 1 - H(\phi_p) & \text{if } B^{h_2}(p) = 0 \end{cases}$$
(9)

Then define the product :

$$\hat{\chi}_{l,h^{1},h^{2}} = \prod_{1 \le p \le N, k \ne l} \chi_{p}.$$
(10)

For a fixed l, each of these products (with respect to h_1 and h_2) is aimed at defining the positive phase $B_+^{l,h^1,h^2} = [B^{h_1}, 1, B^{h_2}]$ and the negative phase $B_-^{l,h^1,h^2} = [B^{h_1}, 0, B^{h_2}]$, where [.,.] is the concatenation operator.

Although these definitions are tedious to write, they provide a convenient notation for implementing a variable number of level set functions in a code. Similar notations have been proposed recently by Bertelli et al (2008).

With these notations, we write the energy that will be minimized in the multi-phase and multi-channel framework. When N level set functions ϕ_l are used, $1 \le l \le N$, let $\Phi = (\phi_1, \ldots, \phi_N)$ and suppose that the heterogeneity measure associated to phase j $(0 \le j \le 2^N - 1)$ is defined by

$$z^{j} = \frac{(u_{0} - c^{j})^{2}}{K(u_{0})^{2}}$$
(11)

where $K(u_0) = \max_{y \in \Omega} u_0(y) - \min_{y \in \Omega} u_0(y)$ is the contrast of u_0 and permits us to have $z^j \in [0, 1]$ and c^j is the mean of phase j. Let also $z = (z^1, \ldots, z^{(2^N-1)})$, then the energy writes:

$$F(z,\Phi) = \nu \sum_{l=1}^{N} \int_{\Omega} |\nabla H(\phi_l)| \, \mathrm{d}x + \sum_{j=0}^{2^N-1} \int_{\Omega} z^j \chi_j \mathrm{d}x$$
(12)

To minimize this expression, we embed the Euler-Lagrange equations in a dynamical scheme with artificial time. Given the initial functions $\phi_l(O, x)$ for $1 \le l \le N$ and $x \in \Omega$, we compute z and update ϕ_l as follows:

$$\frac{\partial \phi_l}{\partial t} = \delta_{\epsilon} \left\{ \nu \operatorname{div} \left(\frac{\nabla \phi_l}{|\nabla \phi_l|} \right) - \sum_{\substack{0 \le h_1 \le 2^{l-1} - 1\\ 0 \le h_2 \le 2^{N-l} - 1}} \left[z(\hat{B}^{l,h_1,h_2}_+) - z(\hat{B}^{l,h_1,h_2}_-) \right] \hat{\chi}_{l,h^1,h^2} \right\} \quad (13)$$

However, a close look at the form of these evolution equations indicates a problem that needs to be fixed: for each level set function, the decision to increase or decrease its value at a given point is taken under the assumption that the sign of the other functions remains unchanged at this point. This results in only one alternative phase being explored for each level set function, which covers all the cases in the 2phase model, but not in the multi-phase model. Hence, for a



Fig. 2 Current evolution of the contours corresponding to the zero level of two functions ϕ_1 (green) and ϕ_2 (red), which aims at segmenting the objects of the image (disk and triangle).



Fig. 3 Diagram of phase transition in the 4-phase model.

point that currently belongs to a given phase, only N alternative phases are explored (one per level set function) out of the $2^N - 1$ other phases. As a consequence, a pixel which is not currently classified in the optimal phase cannot always be moved to the best phase (i.e. the phase whose associated mean value is the closest to the value of the pixel).

An illustration is provided in Figure 2, where the point P, at the top of the triangle is currently in the phase $\{\phi_1 \ge 0, \phi_2 < 0\}$ (that we will denote phase (1, 0) to simplify the notations) and should ultimately be part of the phase (0, 1), which currently best approximates the triangle. However, since P is in $\{\phi_2 < 0\}$, the algorithm based on Equation (5) only chooses between the current phase (1, 0) and the phase (0, 0), which may not lead to the optimal partition.

A symmetrical situation is likely to occur with the evolution of ϕ_2 , which fixes ϕ_1 . Therefore, the algorithm can only drive the evolution of ϕ_2 at point P to the phases (1,0)and (1,1). The only possibility for the point P to be part of phase (0,1) is that one of the phases (0,0) or (1,1) leads to a lower energy compared to the current phase (1,0), and therefore may serve as a temporary non-optimal solution, but this cannot be expected in general.

The diagram of Figure 3 summarizes the possible evolutions that drive points from one phase to the others. An arrow between two phases indicates that a point can be moved between the two phases, so long as this decreases the global energy. Thus, in the 4-phase case, each phase only "sees" two out of the three other phases and 2-step transitions are only authorized in case the first transition decreases the energy, which is not guaranteed. The 8-phase model suffers even more severely from this problem. For a point that is



Fig. 4 Diagram of phase transition in the 8-phase model.



Fig. 5 Single channel test image with eight distinct regions with artificial colors (a). Segmentation obtained by minimization of the energy with the dynamical scheme derived from Euler-Lagrange equations (b). Zero-level of initial level set functions (c). The patterns in the segmentation correspond to pixels of the initial configuration which could not be moved because of the "hidden phase" problem.

currently part of a given phase, the evolution of the level set functions can only drive this point to three other phases, leaving four phases not directly accessible. Hence the diagram of phase transitions for eight phases takes the form of Figure 4. In this case, evolving the level set functions so as to pass a point from one phase to another may require up to three steps, which must all successively decrease the energy. The existence of such a path between the phases becomes very unlikely and the final segmentation highly depends on the initialization.

The consequence of the "hidden phase" problem is the presence of patterns in the final segmentation, created by the points which could not be moved to the appropriate phase, like in Figure 5. These patterns are reminiscent of the initialization, since they are formed of pixels which could not be correctly classified from the very beginning of the iterations. The solution that consists in increasing the regularization parameter to get rid of those disks is not acceptable because it would likely erase small objects from the segmentation (see for example the blue disk inside the triangle). Therefore, we need to find ways to authorize more transitions between phases to avoid this type of local minima.

3.2 Solutions for the hidden phase problem

The previous analysis demonstrates that the evolution of the level set functions at a given point, which is determined according to the comparison of heterogeneity measures in different phases, should rely on a comparison of all phases. The only work that we have found so far, discussing this problem, is the Ph.D. thesis (Hernandez, 2004). We will first expose this solution to the problem and later propose an alternative solution.

3.2.1 Direct choice of the best phase

In Appendix A.3 of his Ph.D thesis, Hernandez has identified the aforementioned problem of "hidden phase" and proposed that the whole set of phases be compared to decide for the evolution of the level set functions at a given point x, instead of only a subset of phases. In the case M = 2 (4phase segmentation), the data force of the original scheme takes the form

$$\begin{aligned} \frac{\partial \phi_1}{\partial t}(x) &= \\ &- \left[((u_0(x) - c^{1,1})^2 - (u_0(x) - c^{0,1})^2) H(\phi_2(x)) \right. \\ &+ \left((u_0(x) - c^{1,0})^2 - (u_0(x) - c^{0,0})^2) (1 - H(\phi_2(x))) \right], \\ \frac{\partial \phi_2}{\partial t}(x) &= \\ &- \left[((u_0(x) - c^{1,1})^2 - (u_0(x) - c^{1,0})^2) H(\phi_1(x)) \right. \\ &+ \left((u_0(x) - c^{0,1})^2 - (u_0(x) - c^{0,0})^2) (1 - H(\phi_1(x))) \right] \end{aligned}$$
(14)

which relies on the comparison of only two terms, once it has been decided whether the point x belongs to the set of points that verify $H(\phi_2) = 1$ (resp. $H(\phi_1) = 1$) or to $1 - H(\phi_2) = 1$ (resp. $1 - H(\phi_1) = 1$).

The idea is then to explore all the phases and to drive the level set functions to the one whose associated mean is closest to the value $u_0(x)$. Hence, equation (14) becomes

$$\frac{\partial \phi_1}{\partial t}(x) = \min\{(u_0(x) - c^{0,0})^2, (u_0(x) - c^{0,1})^2\}
- \min\{(u_0(x) - c^{1,0})^2, (u_0(x) - c^{1,1})^2\},
\frac{\partial \phi_2}{\partial t}(x) = \min\{(u_0(x) - c^{0,0})^2, (u_0(x) - c^{1,0})^2\}
- \min\{(u_0(x) - c^{0,1})^2, (u_0(x) - c^{1,1})^2\}$$
(15)

As before, each evolution equation explores the possibility to increase or decrease the value of the level set function, but this time, all the phases are taken into account to make the decision. The scheme then permits to solve the "hidden phase" problem, for it forces the evolution of the level set function so as to reach the same sign it has in the best phase. Indeed, results indicate that no patterns related to the initialization are present in the final segmentation anymore. However, this method falls in another trap, due to the fact that these possible moves take place in a level set framework. More specifically, we are not in presence of a method of descent anymore. If a pixel goes from one phase to another one, this transition is accomplished by locally changing one or several level set functions until their signs correspond to the best phase. However, if two or more level set functions are meant to change sign, these changes do not generally occur at the same time and the pixel may be directed to a nonoptimal phase, at least temporarily. Therefore, in order to avoid misclassification, which may occur even in the phase associated with the largest heterogeneity measure, it is especially important that convergence is reached at the end of the iterations. Unfortunately, these temporary errors also change the mean of the phases, thus provoking new changes in the system and, in turn, potential new temporary misclassifications. This implies that convergence is hard to reach and that there usually remain a few points of the image that are visibly misclassified.

Since in the original method, based on the Euler-Lagrange descent, any further iteration can only decrease the energy, we can now see how to combine this property, which is very important in the level set framework, with the possibility to 'see' all the phases over the course of the iterations.

3.2.2 Permutations of the phases

In the original multi-phase ACWE, we stated in Section 2.1 that the number of phases in which a pixel can be moved is exactly equal to the number N of level set functions. Any other move may be impossible if none of the N accessible phases decreases the energy. However, we have some latitude regarding what combination of positive and negative parts of the level set functions is used to define a given phase. If we look at the scheme of Figure 3, we see that transitions of pixels from phases (0,0) to (1,1) and from phases (1,0)to (0, 1) may be impossible. The idea is then to permute any couple of phases which are in direct relation with each other, e.g. (1,1) and (0,1). Here, we change the sign of ϕ_1 only in the region $\{\phi_2 \ge 0\}$ so as to perform the permutation. In order to keep smooth level set functions, the permutation is immediately followed by a reinitialization of both level set functions.

Permutation is applied after a fixed number of iterations S. During the first S iterations, the original multi-phase ACWE iteration is performed, until the contours have evolved enough to let patterns due to the "hidden phase" problem appear. This generally does not take more than a few iterations (less than ten), and this parameter, that we will call a cycle of permutation, is denoted by S in this work. Then, we perform the permutation and reinitialization and carry on the ACWE optimization for another S iterations. In the 4-phase model, there is generally no need to compute other iterations and perform other cycles of permutations because all the transitions that were impossible during the first S iterations are



Fig. 6 Energy evolution over 10 cycles of 10 iterations and permutations for the segmentation of a brain MRI image.

realized during the next S iterations, after permutation of two of the phases. However, we performed two cycles of the permutation σ , which permutes the phases (1, 1) and (0, 1), specifically by applying σ two times along 30 iterations (S = 10).

The advantage of the proposed permutation method is that we remain in the exact same framework as the original multi-phase ACWE, except that the solution obtained before permutation is considered as the initialization for the next S iterations, which are computed after permutation and reinitialization. This implies that at each iteration, we are guaranteed to decrease the energy, meaning that a pixel can never be classified, even temporarily, in a phase whose associated mean value is worst than before. This property is illustrated experimentally in Figure 6 for the segmentation of a brain MRI with the plot of a measure of convergence corresponding to energy differences between iterations.

Therefore, after a few dozens iterations, there will not be any risk of gross misclassification, as it is the case with the previous method of Hernandez. The reinitialization is only performed every S iterations, which does not add significant computation burdens. Yet reinitialization helps for the flexibility of the phases, which can change more easily and quickly. Finally, we think that the somewhat slower process of classification of pixels in phases than in Hernandez' method helps the minimization process investigate the histogram of the image. Empirically, we obtained relevant phases with our method, even when the 'direct choice of best phase' method of Hernandez gave poor results (see Figure 7 for results).

The proposed permutation method can be implemented for eight phases as well, though it becomes a little more complex. Again, with N = 3 level set functions, a given pixel can only be moved in three other phases. There remain four other phases, that can be reached by applying different permutations. A possible way to have each phase "see" any of the other in the course of the iterations is detailed now.

First, in the diagram of Figure 4, we optimize the inner and outer squares separately, as in the 4-phase case, by permuting phases (1, 1, 0) and (0, 1, 0) for the inner square

and (1, 1, 1) and (0, 1, 1) for the outer square. At this point, after S iterations performed before the permutation and S iterations performed after, all the transitions between phases $(\cdot, \cdot, 0)$ and between phases $(\cdot, \cdot, 1)$ are performed and there only remains to establish links between the first and second set of phases (inner and outer square).

For a phase of the inner square, there is only one phase of the outer square that is currently accessible. Therefore, applying a circular permutation on inner square phases will permit all of the inner phases to be directly linked to another outer phase and if we apply the same circular permutation three times, all outer phases will have been "seen" by any of the inner phases. Therefore, we need 3S iterations to perform those three circular permutations and the total amount for the whole set of permutations needed in this case is 5S. In our experiments we observed that S can be lowered to three without any significant loss in comparison to S = 10in the 4-phase case. The total number of iterations is thus unchanged compared to the 4-phase case, with 30 iterations (we applied two cycles of permutations, that is $2 \times 5S$ iterations). Overall, the total number of iterations with the proposed permutation method is not significantly different from the number of iterations required with the original implementation of Chan and Vese that suffers from the "hidden phase" problem.

The proposed permutation method could be generalized for N > 3 but we do not detail it, since eight phases are generally sufficient for most multi-object segmentation tasks, as in brain imaging.

3.3 ICM

We now briefly describe the ICM-based approach. The idea is to replace the level-set technique of the previous section by the ICM algorithm, which locally minimizes the energy, yielding a partition of minimal energy when the mean values c^1, \ldots, c^P are fixed. The means are then updated like in the level set approach. Note that contrary to the level-set approach, P is not required to be a power of 2. The ICM has been originally presented in the seminal work of (Besag, 1986). This algorithms relies on a Markovian discrete framework (Winkler, 2006). In other words, unlike the approach that consists in discretizing the Euler-Lagrange equation, one directly considers a discrete version of the energy (1). We first introduce this Markovian point of view (Winkler, 2006) before presenting the ICM algorithm.

We assume that images are defined on a discrete regular grid \mathcal{V} that should be seen as a spatial sampling of the continuous domain Ω . Any site $s \in \mathcal{V}$ corresponds to a point in \mathbb{R}^n . Instead of looking for a separation curve to define the labeling, we consider an image u that takes value in a discrete set of labels, which we identify with the set of the current means of the computed regions $\mathcal{L} = \{c^1, \ldots, c^P\}$. In other words, each pixel is assigned to one possible label, which in turn corresponds to a mean value. We denote by u(s) the value of the label image u at the site $s \in \mathcal{V}$.

For an image u_0 , we represent the first term that corresponds to the homogeneous term of (1), i.e., $\sum_{j=1}^{P} \int_{R_j} (u_0 - c^j)^2 dx$. Since we consider a discrete framework, the integration over a continuous region is replaced by a discrete sum over the sites. For $j = 1, \ldots, P$, let $\mathcal{V}_j = \{s | u(s) = c^j\}$. Thus this term rewrites as the following:

$$\sum_{j=0}^{P} \sum_{s \in \mathcal{V}_j} \|u_0(s) - u(s)\|^2 \quad .$$
(16)

We now need to define a discrete version of the regularization term that consists of a weighted perimeter of the discontinuities. This regularization is known as the Potts prior in the Markovian community (Winkler, 2006). For this purpose, the grid is endowed with a neighborhood system denoted by $\cdot \sim \cdot$. In this paper, we only consider pairwise interactions, i.e., interactions of the form (s, t) with $s \sim t$. We denote by \mathcal{E} the set of all interactions. The regularization term is a sum of adjacent pixels that have different values, i.e., we have:

$$\frac{1}{2} \sum_{(s,t)\in\mathcal{E}} w_{st} R(u(s), u(t)) \quad , \tag{17}$$

where the coefficients $w_s t$ are some non-negative coefficients and where R(u(s), u(t)) = 0 if u(s) = u(t) and 1 otherwise. The coefficient $\frac{1}{2}$ comes from the fact that a discontinuity is counted twice.

In this paper, the 8-connectivity, i.e., the nearest and second nearest neighbors, is considered for the 2D case and the weights w_{st} are set to 1 for the nearest neighbors and to $\frac{1}{\sqrt{2}}$ for the diagonal ones (second nearest ones). In 3D, the 26connectivity is considered and the weights w_{st} are respectively set to 1, $1/\sqrt{2}$ and $1/\sqrt{3}$ for the first, second and third nearest neighbors. We also refer to (Chambolle and Darbon, 2008) for other definitions of discrete perimeter of discontinuities.

We now need to optimize for u an energy of the following form that is the sum of the homogeneous term given by (16) and the regularization term given by (17), the influence of which can be set with a parameter ν :

$$E(u) = \sum_{j=0}^{P} \sum_{s \in \mathcal{V}_j} \|u_0(s) - u(s)\|^2 + \frac{\nu}{2} \sum_{(s,t) \in \mathcal{E}} w_{st} R(u(s), u(t))$$
(18)

For this purpose, we describe the ICM algorithm (Besag, 1986). It is a deterministic iterative procedure that decreases the energy by modifying a given current solution one pixel at a time. Given one current solution u and a site $s \in \mathcal{V}$,

let us define the conditional local energy E_s which is the restriction of the original energy $E_s(l)$ to the site s, where $l \in \{c^1, \ldots, c^P\}$, all the other variables $t \in \mathcal{V} \setminus \{s\}$ being fixed, i.e.:

$$E_s(l) = \|u_0(s) - l\|^2 + \frac{\nu}{2} \sum_{t \mid (s,t) \in \mathcal{E}} w_{st} R(u(s), u(t)) \quad . \tag{19}$$

The ICM picks randomly a site s and then changes the label u(s) to the one that minimizes E_s . This operation is iterated until there is no possible label that decreases the energy. The ICM algorithm is described below:

- 1. Start with a current guess u
- 2. Traverse all sites $s \in \mathcal{V}$

(a) Set $u(s) = \underset{l \in \mathcal{L}}{\operatorname{argmin}} E_s(l)$

3. If no change occurred in the last sweeping of the image (or if it is this number is small enough) return *u*, otherwise go to 2.

In our implementation, the value of the means are updated after one sweep, i.e., all pixels have been updated once, and the ICM is relaunched again with these new parameters. The global process stops when the means values do not significantly change over two consecutive runs, which takes about 20 iterations.

The ICM procedure decreases the energy at each iteration. It is known that the obtained solution is not necessarily optimal and depends on the initial guess (which will be discussed later on). Since it aims at changing the label of a site so as to minimize the energy, the ICM does not suffer from the *hidden phase problem* identified in Section 2.1. By picking the best label in terms of energy, it is close to the algorithm proposed by Hernandez (see Section 2.2.1). However, avoiding the level set framework permits us to remain with a descent method, which guarantees convergence, unlike the approach of Hernandez.

3.4 Results and comparison

We propose several criteria to compare the segmentation results from the multi-phase ACWE and the ICM: computational times, relevance of the segmented regions (qualitative analysis) and the value of the energy, that should be as low as possible. We implemented both algorithms with Matlab, in a matrix implementation which is much faster than iterating on the pixels. The multi-phase ACWE took about 3 minutes for one image (for 30 iterations, see Section 2.2) while the ICM algorithm took about 5 seconds. This large difference can be explained by the simplicity of the latter algorithm and the fact that more iterations must be computed with the multi-phase ACWE to solve the hidden phase problem with permutations. The initalization used with the multiphase ACWE consists of N set of cylinders which define the zero level of each of the N level set functions. A distance function is then applied to determine a value for the level set function at each point of the image. This type of initalization is strongly recommended in (Vese and Chan, 2002). For the ICM algorithm, we used an initialization based on the histogram. We first determine the minimum and maximum of the image u_0 and decompose the obtained interval into P classes which all contain the same number of pixels. We point out here two facts regarding the influence of the initialization on the ICM:

- For brain MRI segmentation, the ICM converges to similar results when initialized with average values derived from the proposed histogram-based method and from the cylinders used with the ACWE method (corresponding to almost identical average intensity values for each phase). Initialization with the histogram based method remains more natural and straightforward to implement for the ICM.
- If initializing the ICM with the average intensity values identified with the ACWE segmentation result, the ICM converges to a segmentation similar to the ACWE rather than to the result obtained with the two other initialization methods. This confirms the influence of the initialization on the ICM result but also emphasizes the fact that both ICM and ACWE converge to different local minima.

From a qualitative point of view, we expect a segmentation method based on a region data term to yield phases whose associated means are representative of the means of the perceived objects. Results presented in Figure 7 for one T1-weighted MR image compare the segmentations obtained with the multi-phase ACWE (top row) and the ICM (bottom row). Each column is associated to a different regularization parameter ν , changing the energy of Equation (1), which we try to minimize with these two methods.

Regarding the ACWE results, we observed a remaining drawback in the lack of flexibility concerning phase evolution. Indeed, once a given structure is represented by several phases, or once phases become empty, it becomes extremely unlikely that these phases will evolve so as to represent another structure. Hence, for each segmentation performed with the multi-phase ACWE, only six phases are actually used to represent the brain and the skull. For some cases, one phase is not used at all and for others, two phases are used to represent the background. This fact is observed in Figure 7, on the top row, where white matter is represented by the same phase as the small hyperintense area present in the original image, corresponding to a blood vessel injected with gadolinium. This vessel structure is recovered in a single phase with the ICM, leading to a qualitatively better segmentation of the image (bottom row). We also applied the two methods to a brain dataset taken from



Fig. 7 2D segmentations obtained for one slice of T1-weighted brain MRI (SPGR). Top row shows results obtained with the multi-phase ACWE (eight phases) of Chan and Vese, free of the hidden phase problem, for different values of the regularization parameter. Bottom row shows the same results, obtained with the ICM, also with eight phases. The same colormap is used for all segmentations, from black for the label corresponding to the lowest mean value (label one) to white for the label corresponding to the highest mean value (label eight).

the Internet Brain Segmentation Repository¹. While no significant differences between the results provided by the ICM and the multi-phase ACWE could be observed when processing one or only few slices, the ICM demonstrated better sensitivity when the whole MRI volume (116 slices) was processed. In particular, all of the gray nuclei were still identified individually on ICM segmentations with six and eight phases, whereas the putamen could not be detected by the multi-phase ACWE (see Figure 8). Our method therefore improves the limitation observed in a statistical framework by (Rajapakse and Kruggel, 1998), who reported that some of the subcortical gray structures were missed with their algorithm based on the ICM. Hence, ICM provides us with a good overall segmentation, which could be further refined by introducing gradient information or spatial relationships to work on one or several specific structures.

In addition, the multi-phase ACWE and the ICM can be compared on the basis of the final energy level, to judge the performance of the optimization processes. Table 1 provides final energy values for all experiments presented in Figure 7. These results indicate that for small regularization weights, the two methods yield equally good results, with a small difference in favor of the ICM. For larger weight values, the regularization obtained with the ICM achieves better energy minimization, as can also be seen on the precision of anatomical segmentation in Figure 7.

¹ The MR brain data sets and their manual segmentations were provided by the Center for Morphometric Analysis at Massachusetts General Hospital and are available at http://www.cma.mgh.harvard.edu/ibsr/.



Fig. 8 Original data taken from IBSR (a) and 3D segmentations obtained with the multi-phase ACWE with eight phases (b), ICM with eight phases (c) and ICM with six phases (d). The whole volume (116 slices) was processed here.

Since both methods are known to only converge to a local minimum, we also have implemented the Metropolis sampler embedded into a simulated annealing algorithm. We do not detail the Metropolis sampler with SA here since it only differs from the ICM in that new labels are generated randomly and the transition is accepted if the energy is decreased. If the energy does not decrease, the transition can still be accepted with a probability $\exp \frac{-\Delta E}{T}$, where T is the temperature parameter. We refer the reader to (Geman and Geman, 1984; Kirkpatrick et al, 1983; Winkler, 2006) for further details. In our implementation, T was initially set to 0.1 and multiplied at each iteration by a constant set to 0.99. Note that this geometric scheme for temperature does not guarantee the convergence of the process towards a global minimizer anymore (see (Winkler, 2006) for further details) but can at least give an approximation of the global solution. We stopped after 5000 iterations, which took about one hour, also for eight phases. Although the computational cost of this algorithm makes it hard to use in practice, it provides a good estimate with regard to the global minimum of the energy. An example of whole brain segmentation performed with SA on Brainweb² data is shown in Figure 9 . From this comparison, we find that the multi-phase ACWE and ICM perform well, as they reach local minima whose energy is close to that computed with SA, especially when the regularization parameter ν is low. For $\nu = 0.1$, which corresponds to a highly regularized segmentation, the multi-phase ACWE performs poorly, notably by decomposing the background in two phases in Figure 7. Although we obtain sig-



Fig. 9 Brainweb data set (a) with 5% SNR and segmentation, performed by optimizing the energy of (Eq.1) with the simulated annealing algorithm.

	$\nu = 0.003$	$\nu = 0.005$	$\nu = 0.01$	$\nu = 0.1$
ACWE	50.03	65.20	111.27	721
ICM	42.66	57.94	96.12	589
SA	42.64	57.83	94.06	516.89

Table 1 Comparison of the values of the energy (12) for the optimization processes obtained with the multi-phase ACWE, the ICM algorithm and the simulated annealing (SA) algorithm.

nificantly higher energy with ICM than with SA in this case, compared to the cases with smaller ν , the energy computed with ICM is still only 14% larger than the final energy level computed with SA. This comparison therefore demonstrates that, in spite of poor performances in other applications (see the recent paper of (Szeliski et al, 2008) for energy minimization benchmarks in stereo, image stitching, interactive segmentation, and denoising), ICM can be successfully used in multi-phase segmentation based on the Mumford-Shah energy.

From these results, it appears that the minimization of energy (12) can be best performed with the discrete approach proposed by the ICM algorithm, rather than by the continuous approach of the level set based ACWE.

4 Extension to the multi-channel case

At this point, we evaluated two methods to minimize the energy (1) and to obtain a relevant multi-phase segmentation of an image. We are now interested in the problem of finding objects according to the information provided by several channels. For this purpose, we will first justify our choice for a fusion operator on heterogeneity measures, considering logic-based rules to fuse information from different image channels. We will then present the *logic framework* proposed in (Sandberg and Chan, 2005), in the binary (2-phase) case. Finally, we will extend our multi-phase segmentation model to integrate multi-channel information. One of the main difficulties for doing so is to avoid conflictive objectives, which lead to contingent results. We will therefore discuss the possible rules of fusion from a semantic point of view.

² The Brainweb data sets are available at http://www.bic.mni.mcgill.ca/brainweb/.

4.1 Fusion operators for merging heterogeneity measures

The first question to be addressed is the selection of the most relevant level for fusing information from different channels. These possible levels of fusion can be classified into three categories: the data (fusion of the different channels), the segmentation (fusion of segmentations obtained independently for the different channels) and the fusion of a quantity that is used during the segmentation process. Arguments against the first two possibilities can be found in (Sandberg and Chan, 2005), and it is therefore suggested to achieve the fusion on the heterogeneity measures, which are defined for each point $x \in \Omega$ by:

$$z_i^j(x) = \frac{|u_0^i(x) - c_i^j|^2}{K(u_0^i)^2}$$
(20)

where i = 1, ..., M denotes the channel, $K(u_0^i)$ is the contrast of u_0^i (see Equation (11)) and j denotes the phase.

Since we have to combine heterogeneity measures defined as values in [0, 1], we propose to rely on the fuzzy fusion framework for choosing the most appropriate fusion operators. Indeed, fuzzy sets and possibility theory offer a large spectrum of combination operators (Dubois and Prade, 1985; Yager, 1991), with well established properties and behaviors, and that allow adapting the operator to each particular problem at hand. Among the main operators, we find t-norms, t-conorm, mean operators, symmetrical sums, and operators taking into account conflict between sources or reliability of the sources. A synthesis can be found in (Bloch, 1996; Dubois et al, 1999; Bloch, 2008). Moreover, criteria for choosing an operator have been described in the literature and are of great help when facing a concrete segmentation task.

A classification of these operators with respect to their behavior (conjunctive, disjunctive, or compromise), the possible control of this behavior, their properties and their decisiveness has been proposed in (Bloch, 1996) and proved to be useful for several applications in image processing. The first class consists of operators which have always the same behavior, whatever the values to be combined. This class can be further refined into:

- conjunctive operators, i.e. that provide a result always smaller than each of the values to be combined. Typical examples are t-norms (that extend intersection to fuzzy sets), which are commutative, associative and monotonous operators, and have 1 as unit element; the most used ones are the minimum, the product, or Lukasiewicz t-norm $(t(z_1, z_2) = \max(0, z_1 + z_2 - 1));$
- disjunctive operators, that provide a result larger than each of the values to be combined. A representative class is composed of t-conorms (fuzzy union), which are commutative, associative and monotonous operators, and have 0 as unit element; the most used ones are the maximum,

the algebraic sum, or Lukasiewicz t-conorm $(T(z_1, z_2) = \min(1, z_1 + z_2));$

- compromise operators, that provide a result comprised between the minimum and the maximum of the values to be combined, a typical example being the arithmetical mean, or more generally mean operators of the form $(\frac{z_1^{\alpha}+z_2^{\alpha}}{2})^{1/\alpha}$. The parameter α is used to tune the strength of the operator, which tends towards the minimum for $\alpha = -\infty$ and towards the maximum for $\alpha = +\infty$ (the value $\alpha = 1$ provides the classical arithmetical mean). These operators are not associative but they have the strong property of being idempotent.

The idempotence property for the fusion operator f (i.e. f(x, x) = x) is important when sources are considered as being independent, i.e. there is no reason to reinforce the information they provide (Dubois et al, 1999) (as would be the case with most t-norms, which weaken the combined values, or t-conorms, which augment them). An interesting parallel can be drawn with the statistical independence property, which leads to express the conditional probabilities as a product over the sources (i.e. a sum when taking the logarithm to provide an energy function).

For multi-phase segmentation, conjunctive combinations of heterogeneity measures on a phase j will perform a union of observations. In this case, the fused heterogeneity measure is lower than the minimal heterogeneity measure on the different channels, which guarantees that any area homogeneous with the current average value of phase j, in at least one channel, can be attached to the phase. On the other hand, disjunctive combinations applied to a phase j favor the intersection of the observations of an object across the channels. In this case, all heterogeneity measures must be low to attach a point to the phase j.

If idempotence is besides required, then the only possible choice is the minimum t-norm or maximum t-conorm (the only idempotent operators in these classes).

Another class that could be of interest consists of operators that have a variable behavior, depending on the values to be combined. This is the case for some symmetrical sums for instance, such as $\sigma_0(z_1, z_2) = \frac{z_1 z_2}{1-z_1-z_2+2z_1 z_2}$ or $\sigma_+(z_1, z_2) = \frac{z_1+z_2-z_1 z_2}{1+z_1+z_2-2z_1 z_2}$. For instance σ_0 behaves disjunctively for high values $(z_1 \ge 1/2 \text{ and } z_2 \ge 1/2)$, conjunctively for small values $(z_1 \le 1/2 \text{ and } z_2 \le 1/2)$ and as a compromise otherwise. This leads to an increase in the dynamics of the combined values. On the contrary, σ_+ tends to decrease the dynamics.

Finally the third class consists of operators depending also on some global information such as reliability of each source, or conflict between sources. This can be useful when sources cannot be considered as equivalent, for instance because one image is more reliable than another one, either globally or for specific classes or structures. In the present work, we mostly discuss applications in MRI imaging, for which sources can be considered equivalent. Since we are looking for fusion rules that can be easily interpreted from a semantic point of view, we will only use conjunctive and disjunctive operators (i.e. union and intersection operators). For the specific choice of the operator, the idempotence property is also adapted to our case and we will therefore focus on the maximum operator for heterogeneity measures, to enforce intersection of channels information and on the minimum operator, to achieve the union.

4.2 The binary case

The logic framework of (Sandberg and Chan, 2005) was designed to catch an object in a 2-phase framework, according to several possible set operations, applied on multiple channels. For two channels, if O_1 and B_1 are the observations of the object and the background in the first channel, and O_2 and B_2 , are the observations of the object and the background in the second channel, the final region of the object O can be defined as either $O_1 \cup O_2$ or $O_1 \cap O_2$.

The fusion of the z_i^j with respect to the different channels can be defined with a union function, that considers that a point belongs to a given phase as long as the heterogeneity measure is low in at least one channel. An example of such union function (given here for only two channels) is:

$$f_{\cup} = (z_1^j \cdot z_2^j)^{1/2}.$$
(21)

An intersection function can be defined as well, that considers that a point belongs to a given phase only when all heterogeneity measures are low. The intersection function proposed in (Sandberg and Chan, 2005) is:

$$f_{\cap} = 1 - ((1 - z_1^j)(1 - z_2^j))^{1/2}.$$
 (22)

It should be noted, however, that the operators proposed in Equations (21) and (22) are actually compromise operators, according to the classification of Section (3.1). For instance f_{\cup} is a mean operator obtained for $\alpha = 0$ (geometrical mean). Although it provides a value that is smaller than the arithmetical mean, it is still not a true conjunctive operator.

Depending on the type of segmentation to perform, different combinations of the two fusion rules are proposed. For example, the union of all observations of an object Acan be obtained by applying a union function (conjunctive operator) on the heterogeneity measures associated to the phase that contains A (say phase 1) while the heterogeneity measures associated to the other phase are fused according to the intersection rule (disjunctive operator). Here, the different possibilities rely upon De Morgan's law, which states that for two sets A_1 and A_2 , $(A_1 \cap A_2)^c = A_1^c \cup A_2^c$. Enforcing this law is required to avoid *conflictive objective* in the segmentation.

In the level set ACWE formulation, the objective function corresponding to the union of all observations applied on phase 1 (and therefore the intersection of all observations applied on phase 2) writes

$$F(\overline{c^{1}}, \overline{c^{2}}, \phi) = \nu \int_{\Omega} \delta(\phi) \mid \nabla \phi \mid \mathrm{d}x + \int_{\Omega} f_{\cup}(z_{1}^{1}, z_{2}^{1}) H(\phi) \mathrm{d}x + \int_{\Omega} f_{\cap}(z_{1}^{2}, z_{2}^{2})(1 - H(\phi)) \mathrm{d}x.$$

$$(23)$$

Other combinations can be derived to obtain the segmentation of the intersection or union of all observations A_i of a given object or other possible bitwise logic operations, such as $A_1 \cap \neg A_2$.

Therefore, this framework can handle the problem of multi-channel segmentation in a 2-phase framework, in the sense that conflictive objective are avoided, but needs to be adapted to the multi-phase case. We now present and discuss possible rules of fusion from a semantic point of view.

4.3 Multi-phase case

In order to obtain a relevant multi-phase segmentation of a multi-channel image, several elements must be taken into account. First, as in the 2-phase case, conflictive objective must be eliminated. Therefore, we must avoid assigning a point to several phases at the same time or, reciprocally, a point to be put away of all the existing phases. This constraint of avoiding conflictive objective is fundamentally related to the fact that our segmentation method ultimately results in a partition of the image: no hole or overlap is possible and an arbitrary decision is made automatically, in case of conflictive objective that would cause a hole or overlap otherwise. In the 2-phase case, we have seen that conflictive objectives are avoided by assigning to the two phases different fusion operators, namely a conjunctive and a disjunctive operator. In the multi-phase case, we need to explore what the possibilities are to assign a given type of fusion operator to a phase. It is clear for example that having two phases segmented with a union fusion rule between heterogeneity measures is likely to lead to a conflictive objective, even if intersection fusion rules are used on all other phases. This is illustrated in Figure 10, where two objects and a background appear with a different shape in the two channels. Three phases (at least) are required here, two for the objects and one for the background. While the conflicts between the objects and the background would be solved by the duality of the union and intersection fusion rules (respectively



Fig. 10 Two-channel image illustrating the possible conflict created by the various appearances of the objects 1 and 2 in the two channels. Some parts of the image (point x for example) are inside object 1 in the first channel (left) and inside object 2 in the second channel (right). If both objects are segmented as a union through the channels, no clear decision is made regarding the conflictive part and a somewhat arbitrary decision will be taken to enforce partition constraint.

for the two objects and for the background), the conflict between the two objects is not resolved. Here, the two phases dedicated to the objects would both tend to take the conflictive part in charge but the partition constraint would not allow this. An arbitrary decision is then made, which may, or may not, serve a given purpose.

A second element that must be taken into account is the semantic relevance of the segmentation. In addition to obtaining results that are not contingent (i.e. strongly relying on initialization, or on small differences in gray levels), we expect the segmentation to be easily interpretable and to isolate objects that can be perceived or easily predicted when we look at the data. This semantic relevance element leads to a surprisingly drastic conclusion, namely that we should completely avoid union fusion rules (conjunctive operators). To illustrate this point, we propose a synthetic example in Figure 11 (a). The multi-channel image is composed of three distinct regions in each channel, with a common background and the central rectangle that decomposes into two sub-objects in the two channels. Formally, we can distinguish four regions according to their values in channel 1 and 2 (written as a couple): (10, 10), (0, 100), (0, 0) and (100, 0) (see Figure 11 (a), bottom row). In a 2-phase segmentation, we tried different initializations to take different parts of the central rectangle in charge in phase 1, the second phase being mostly background. We used a union fusion rule for the phase dedicated to the object (phase 1) and an intersection rule for the phase dedicated to the background (phase 2), following the recommendation of (Sandberg and Chan, 2005). We then expect that the phase dedicated to the object will expand, due to new areas which are homogeneous with the current phase 1 in at least one channel, and become stable.

However, surprisingly, all these strategies invariably lead to the same result, shown in Figure 11 (b). Although, in the course of the iterations, the part (0,0) has been classified in phase 1, it is ultimately part of the background, in phase 2. This phenomenon has a clear explanation, in that using



Fig. 11 (a): Two-channel test image (first and second row) and image with the four distinct regions (third row). White is 100, black is 0 and gray is 10. (b): Results of 2-phase segmentation with different initializations and a union fusion rule inside the contour (phase 1), intersection fusion rule outside the contour (phase 2).

a union fusion rule permits one to combine very heterogeneous parts of the image in the same phase, therefore leading to means which are not representative of any of the actual parts of the phases. Since our classification relies on a distance with the mean of the segmented regions, changes can occur when the mean is too distorted, as is the case in our synthetic example. Unfortunately, this makes the interpretation of the combined multi-phase multi-channel segmentation very hard. Indeed, we would not expect that parts that have been added to a phase, segmented with a union fusion rule, could be later put into another phase. When we initialize phase 1 with the points of values (100,0), it is puzzling that the final result attaches the part (0,100) rather than (0,0), the latter being homogeneous with the initial phase 1 in one channel and not the former. Our conclusion is that union fusion rules lack stability by letting the possibility to introduce great changes in the mean of a phase in a channel, which results in rather unpredictable segmentation results.

We now see that the two constraints on the segmentation process to provide a *partition* of the image data and to enforce *semantic relevance* lead to contradictory conclusions: while the first one prescribes to use intersection fusion rules in combination with union fusion rules, relying on De Morgan's law, the second one suggests to avoid the use of union fusion rule. Indeed, using intersection fusion rule for all the phases leads to a greater stability of the average values within the phases. At the same time, by doing so, we do not explicitly decide how to classify conflictive parts of the image data and therefore, introduce contingency in our results. However, one decisive advantage of the multi-phase case over the 2-phase case is that one can decide the appropriate number of phases. In the case of our test image, we have seen that four regions can be distinguished by their value in each channel. Employing intersection fusion rules in the segmentation process to identify regions that are homogeneous in all the channels, we need four phases to completely decompose the original multi-channel image into objects that are homogeneous in all the channels.

Contrary to the use of union fusion rules, which does not lead to a clear interpretation of the segmented regions (one clear interpretation could be that we segment "regions that are homogeneous in at least one channel", but we have seen with our test image that such a naive interpretation is not applicable), applying an intersection fusion rule in all phases does lead to a clear interpretation, namely, that we segment regions which are homogeneous in all the channels. As in the multi-phase and single-channel case, we must decide on the number of phases that is required for a given problem, but the additional difficulty in the multi-channel case is that this choice is based on what we think is the number of regions that are distinguishable from all the channels. We will discuss the specific example of pathological brain MRI in the next section. For the moment, we formalize our multiphase and multi-channel segmentation method for a fixed number of phases R_j , j = 1, ..., P with $\bigcup_{j=1}^{P} R_j = \Omega$.

The multi-phase segmentation model, posed as a minimization problem in Equation (1), is straightforwardly extended to the multi-channel case, by fusing heterogeneity measures with a disjunctive fusion operator. Using the maximum as fusion operator, the energy to minimize is:

$$E = \sum_{j=1}^{P} \int_{R_j} \max_{i=1,\dots,M} (z_i^j(x)) \mathrm{d}x + \nu \cdot \mathrm{Length}(\partial R_j), \quad (24)$$

where ∂R_j denotes the contour of the phase R_j and M is the number of channels. The heterogeneity measure z_i^j is defined for each channel i with respect to the phase j as in Equation (20).

Finding a minimum for this energy can be achieved by any of the methods discussed in Section 2: the level-set ACWE, the ICM algorithm or the simulated annealing (or any other optimization algorithm that is used to minimize (1)). However, here again, the level-set implementation discussed imposes a number of phases of the form $P = 2^N$, where N is the number of level set functions.

5 Segmentation of pathological brain datasets

In this section, we present some experiments to validate the proposed model of multi-phase and multi-channel segmentation on pathological brain dataset. This model relies on the energy of Equation (24), solved with the ICM algorithm, for the reasons presented in Section 2.4. Three parameters need to be set, namely the number of iterations, the regularization parameter and the number of phases. While the first two parameters could be fixed to 20 iterations and $\nu =$

0.005, we used only five phases in single-channel FLAIR (T2-weighted) segmentation and seven phases in single-channel SPGR (T1-weighted) as well as for combined multi-channel (FLAIR, SPGR) segmentation.

In single-channel image segmentation, T1-weighted MR images have sufficient contrast to extract normal anatomical structures such as WM, GM, CSF and gray nuclei. However, they provide very low contrast between pathological structures (tumor, edema) and GM. On the other hand, while T2-weighted MR images do not permit to clearly distinguish between normal anatomical structures, they provide very good contrast on the tumoral region. We tested both single- and two-channel images, for two clinical cases with a brain tumor (low-grade glioma). Results in Figure 12, for one case, show that the multi-channel segmentation is able to extract the most relevant information from the two MRI protocols: the tumoral region (tumor and edema) is essentially segmented from the FLAIR image while segmentation of the normal brain structures is derived from the SPGR image. Manual segmentation of the tumor on the FLAIR and SPGR data was performed by an expert neurosurgeon, to compare with our automated segmentation results as illustrated in Figure 14 (a) and (c) for single-channel segmentation and in Figure 14 (b) and (d) for multi-channel segmentation. We observed that the contours obtained in the latter case are most similar to the manual contours. Postprocessing of the overall segmentation result was applied to extract the tumoral region, consisting in selecting the connected component corresponding to the tumor and filling holes in it. Hence, the different parts of the red contour in Figure 14(b-d) are actually connected in 3D. Figure 13 and Figure 15 present the same type of results for another patient. The same post-processing was used in Table 2, where we have evaluated our different segmentations of the tumor by comparing them to the manual segmentations in terms of true positive (TP), false positive (FP), similarity index (SI) and Jaccard index (JI)³. This evaluation was performed for the whole tumor in 3D and does not necessarily correspond to what can be seen in the slice displayed in Figure 14. An additional comparison with the method proposed in (Khotanlou, 2008) indicates that our model can compare with a dedicated method which makes use of symmetry analysis and spatial relations, since our SI is only lower than that obtained by Khotanlou by approximately 5%.

Manual segmentations of normal brain structures were not available but we can observe that the segmentation of these structures corresponds to areas which are well differentiated in the SPGR image and that a strong correspon-

³ We recall that for two sets A and M, which correspond here to the automatic and manual segmentations, we denote by |A| the cardinal of A (i.e. the number of voxels) and have the following definitions: $TP = \frac{|A \cap M|}{|M|}, FP = \frac{|A| - |A \cap M|}{|A|}, SI = 2\frac{|A \cap M|}{|A| + |M|}$ and JI = 1

 $[\]frac{|A \cap M|}{|A \cup M|}$



Fig. 12 One slice of SPGR dataset (a) and FLAIR dataset for Case 1 (c). Multi-phase segmentation (ICM) of the SPGR data with seven phases (b) and of the FLAIR data with five phases (d). Multi-phase and multi-channel segmentation with seven phases (e).



Fig. 13 One slice of SPGR dataset (a) and FLAIR dataset for Case 2 (c). Multi-phase segmentation (ICM) of the SPGR data with seven phases (b) and of the FLAIR data with five phases (d). Multi-phase and multi-channel segmentation with seven phases (e).

dence exists between the single- and multi-channel segmentations. Direct comparison between the two phases that represent WM in 3D (no post-processing was performed and we did not fuse several phases) gives a similarity index of 84%.

It is interesting to notice that, in Figure 12 (e), the tumor has been segmented according to what seems to be more like the union of the observations of the pathological area through the channels than the intersection (the similarity index with the intersection of the manual segmentations is 73% and increases to 81% with the union of the manual segmentation). However, this does not contradict what has been exposed in Section 3.3 concerning the semantic relevance of





Fig. 14 Contour of the tumor of Case 1 in single-channel SPGR segmentation in red, compared to the manual SPGR contour in blue (a); contour of the tumor in multi-channel segmentation in red, compared to the manual SPGR contour in blue and the manual FLAIR contour in green, registered on SPGR image (b); contour of the tumor in FLAIR segmentation in red, compared to the manual FLAIR contour in green (c); contour of the tumor in multi-channel segmentation in red, compared to the manual SPGR contour in blue and the manual FLAIR contour in green, registered on FLAIR image (a).

	TP	FP	SI	JI
CASE 1				
SC SPGR/SPGR	72	21	75	60
SC FLAIR/FLAIR	81	9.5	85	75
MC/SPGR	76	27	74	59
MC/FLAIR	77	6	84	73
HK SC FLAIR/FLAIR	84.5	4.6	89.6	81.2
CASE 2				
SC SPGR/SPGR	38	77	28	16.6
SC FLAIR/FLAIR	76	0.5	86	75
MC/SPGR	68	1.45	80	67
MC/FLAIR	83.6	2.6	88	79
HK SC FLAIR/FLAIR	92.2	5.3	93.4	87.6

Table 2 Evaluation of tumor segmentations for two cases, compared to manual segmentations in SPGR and in FLAIR. True positive (TP), false positive (FP), similarity index (SI) and Jaccard index (JI) are all given in percent. Single-channel (SC) segmentations of SPGR and FLAIR data are compared to the manual segmentation of the same image. Multi-channel (MC) segmentations are compared to both SPGR and FLAIR manual segmentations. We also provide a comparison with the work of Hassan Khotanlou (HK SC FLAIR/FLAIR) with a dedicated method applied on the same cases in FLAIR.



Fig. 15 Contour of the tumor of Case 2 in single-channel SPGR segmentation in red, compared to the manual SPGR contour in blue (a); contour of the tumor in multi-channel segmentation in red, compared to the manual SPGR contour in blue and the manual FLAIR contour in green, registered on SPGR image (b); contour of the tumor in FLAIR segmentation in red, compared to the manual FLAIR contour in green (c); contour of the tumor in multi-channel segmentation in red, compared to the manual SPGR contour in blue and the manual FLAIR contour in green, registered on FLAIR image (a).

the segmentation. Our segmentation can still be interpreted in terms of regions that are homogeneous in all the channels but this notion of homogeneity is relative and depends on two parameters: the regularization coefficient (if intensity differences are small enough, we tend to decompose the image in as few regions as possible) and the number of phases (if we have more phases, we become more sensitive to small variations of intensity). Hence, in the case of the tumor, given the relatively low number of phases used in the multichannel segmentation (seven, which is the same as in the SPGR segmentation), the intersection fusion rule between heterogeneity measures has led to a tumor phase which is relatively homogeneous in all the channels, although it does not correspond to the intersection of the tumor phases obtained in single-channel segmentations in SPGR and FLAIR. This means that we cannot think of the process of fusion in purely geometric terms from what is obtained in singlechannel segmentations.

6 Conclusion

We have based our segmentation method on an energy minimization model (Mumford-Shah) that permits to obtain homogeneous regions. In order to minimize this energy, we have improved the level set method proposed by Chan and Vese, implemented the ICM algorithm with a regularization based on the Potts prior and compared the energy with that obtained with simulated annealing. Different experiments demonstrated the superiority of the ICM algorithm in terms of relevance of the segmented regions, energy minimization and speed ratio.

Our multi-channel extension is based upon a simple fusion rule that consists in considering the maximum of heterogeneity measures through the channels. We have demonstrated that this rule is part of the only class that carries out a segmentation which can be easily interpreted, and that corresponds to the intersection of the observations of a same object through the channels. In other words, the process results in phases that are relatively homogeneous in all the channels. Finally, we applied our method to multi-protocol (T1 and T2) MRI data with brain tumors, and observed that the most relevant information was used from both protocols, namely, that the SPGR image was used to segment the normal structure, while the FLAIR image serves to delineate the tumor. By comparing our multi-phase and multichannel segmentation of the tumor to manual segmentations, we found similarity indices which were only a bit lower than those obtained with a dedicated method (by about 5%). However, our method leaves us with much more information regarding the rest of the structures of the brain.

By restricting this work to one of the simplest models, we aimed at giving an understanding of the possibilities to fuse multiple sources of information in segmentation. Future works will focus on specific tasks for which prior information such as spatial relations or contrast between structures could be used to constrain the evolution of the phases (see (Colliot et al, 2006) for example). Another interesting question copes with the automation of the decision regarding the number of phases, recently discussed in (Sandberg et al, 2008) or in (Brox and Weickert, 2004), which could still decrease the number of parameters decided by the user.

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