

Low dose x-ray phase-contrast and absorption CT using Equally-Sloped Tomography

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Abstract. Tomographic reconstruction from undersampled and noisy projections is often desirable in transmission CT modalities for purposes of low dose tomography and fast acquisition imaging. However under such conditions, due to the violation of the Nyquist sampling criteria and the presence of noise, reconstructions with acceptable accuracy may not be possible. Recent experiments in transmission electron tomography have shown that the technique of Equally-Sloped Tomography (EST), an exact tomographic method utilizing an oversampling iterative Fourier based reconstruction, provides more accurate image reconstructions when the number of projections is significantly undersampled relative to Filtered Back Projection (FBP) and algebraic iterative methods. Here we extend this technique by developing new reconstruction algorithms which allow for the incorporation of advanced mathematical regularization constraints, such as the Nonlocal Means Total Variational model, in a manner that is consistent with experimental projections. We then evaluate the resulting image quality of the developed algorithm through simulations and experiments at the European Synchrotron Facility on image quality phantoms using the x-ray absorption and phase contrast CT modalities. Both our simulation and experimental results have indicated that the method can reduce the number of projections by 60-75% while achieving comparable or better image quality than the conventional reconstructions. As large-scale and compact synchrotron radiation facilities are currently under rapid development worldwide, the implementation of low dose x-ray absorption and phase-contrast CT could find broad applications in biology and medicine using these advanced x-ray sources.

Index Terms — Equally-Sloped Tomography, Computed Tomography, Iterative Tomographic Image Reconstruction, Total-Variation Nonlocal Means Optimization, X-Ray Phase Contrast CT, CT, Radiation Dose Reduction.

1. Introduction

Tomographic implementation of emerging transmission imaging modalities, such as synchrotron x-ray absorption and phase contrast imaging, present important applications to biology and medicine (Cloetens *et al.*, 1999; Langer *et al.*, 2008; Momose *et al.*, 1996; Nugent *et al.*, 1996; Bech *et al.*, 2009; Weitkamp *et al.*, 2008; Connor *et al.*, 2009). For instance, synchrotron absorption imaging allows for tunable elemental mapping of specimens, while phase contrast imaging offers 100-1000 times increased sensitivities in the hard x-ray region relative to absorption imaging (Cloetens *et al.*, 1999). However, due to the time intensive line scanning of parallel projections and concerns about radiation damage to biomedical specimens, tomographic implementation of these techniques for applications beyond that of material science is problematic. For instance, in the common in-line holography implementation of phase contrast CT, multiple detections (often 3 or 4) at different sample to detector distance are required to recover the phase data for each projection (Langer *et al.*, 2008); this, coupled with the tomographic requirement of multiple projections from many directions, yields the acquisition time impractical for *in vivo* imaging. Moreover, as both modalities utilize the transmission of ionizing radiation, the tomographic requirement of multiple projections gives rise to significant concerns about the radiation dose in potential biomedical applications. Consequently, techniques for both fast acquisition and low dose tomography are necessary.

One method to simultaneously achieve both the objectives of reduction of acquisition time and radiation is to under-sample the number of tomographic projections. However, under this condition, conventional reconstruction algorithms do not yield adequate results due to the violation of Nyquist sampling criteria. Mathematically, the tomographic reconstruction then represents an ill-posed problem due to the presence of missing information, i.e. missing projections, and inconsistencies generated from the presence of noise. In other fields, such as coherent diffraction microscopy where somewhat an analogous missing information problem is encountered due to missing phase data of the diffraction recordings, iterative oversampling Fourier methods have proven to be a powerful technique for solving for missing information under noisy conditions for noncrystalline and nanocrystal samples (Miao *et al.*, 1999; Pfeifer *et al.*, 2006; Abbey *et al.*, 2008; Raines *et al.*, 2010). Since the Fourier slice theorem provides a Fourier relation for tomographic imaging, such methods to aid the recovery missing information in tomographic reconstruction can be exploited. In particular, previously an exact Fourier based oversampling method for tomographic reconstruction, termed Equally-Sloped Tomography (EST), has been introduced (Miao *et al.*, 2005). Both numerical simulations and experimental results from X-ray diffraction and electron microscopy have shown that EST enables more accurate image reconstructions even when the number of projections is significantly undersampled relative to Filtered Back Projection (FBP) and algebraic iterative methods (Miao *et al.*, 2006; Lee and Fahimian *et al.*, 2008; Mao *et al.*, 2009).

The purpose of this work is twofold. First, to further extend the capability of EST in dealing with the problem missing data under noisy conditions, we develop and test a new tomographic reconstruction structure that allow for the flexible incorporation of regularization schemes. In particular, an iterative Fourier-based tomographic reconstruction algorithm is introduced which incorporates the advanced model of Nonlocal Means Total Variational (NL-means) regularization, in a manner that is strictly consistent with experimentally measured projections (Buades *et al.*, 2005, 2006). Second, we implement the method on the absorption and phase contrast x-ray CT modalities, and through a series of experiments and simulations quantify the resulting image quality. We then explore the feasibility of radiation dose and acquisition time reduction through reducing the number of projections by determining the factor by which the number of projections can be undersampled while still producing reconstructions of equivalent or better quality relative to conventional full-dose FBP reconstructions, as measured by such metrics as resolution, signal to noise ratio, and contrast to noise ratio.

2. Theory

2.1 Fourier Formulation

The Fourier slice theorem, which equates the 1D Fourier transform of the projection data at a given angle to a slice of the 2D Fourier transform of the reconstructed image at the same angle, forms the foundation of tomographic reconstruction (Kak and Slaney, 2001; Natterer, 2001). Although the theorem provides an elegant conceptual method for the reconstruction of tomographic data, the theorem's direct computerized application has thus far been problematic due to the fact that interpolations must be implemented in Fourier space since equally-angled acquisitions result in a polar distribution of points in Fourier space, while conventional FFT algorithms utilize a Cartesian point set (Kak and Slaney, 2001; Natterer, 2001). Since it is believed that no direct accurate and fast polar to Cartesian discrete Fourier transform can be constructed in a manner analogous to the continuum polar Fourier transform (Briggs and Henson, 1995), a variety of interpolative methods have been proposed, the most sophisticated of such methods include the utilization of the non-uniform FFT (NUFFT) (Delaney and Bresler, 1996; Matej *et al.*, 2004; Zhang-O'Connor and Fessler, 2006). However, all such methods must utilize some degree of Fourier space interpolation; for example, in NUFFT techniques, the degree of interpolations is prescribed by necessary accuracy parameter that in turn controls the complexity of interpolations (Ware, 1998; Greengard and Lee, 2004).

Recently, the existence of a direct exact fast Fourier transform algorithm and its inverse between a related pseudopolar grid and the Cartesian grid, called the pseudopolar fast Fourier transform (PPFFT), has been proven (Averbuch *et al.*, 2008a; Averbuch *et al.*, 2008b). In contrast to other techniques, the PPFFT consists of 1D operations with a complexity of $O(N^2 \log N)$ comparable to the standard Cartesian FFT, and requires no accuracy parameters or interpolations, resulting in machine accuracy. As shown in Figure 1(a), for a $N \times N$ Cartesian grid, the corresponding pseudopolar grid is defined by a set of $2N$ lines, with each line consisting of $2N + 1$ grid points mapped out symmetrically on N concentric squares. The $2N$ lines are subdivided into a basically horizontal (BH) group defined by $y = sx$, where $|s| \leq 1$, and a basically vertical (BV) group defined by $x = sy$, where $|s| \leq 1$; the BV and BH groups are symmetric under the interchange of x and y . The pseudopolar lines are termed "equally-sloped" since the slope, s , of each successive line as defined above changes by an equal slope increment of $\Delta s = 2/N$ as opposed to the polar grid lines which change by equal-angled increments. The fact that, in contrast to the polar grid, the vertical and horizontal displacements in each group are uniform when the grid is transversed horizontally or vertically, is one reason for the existence of a direct fast Fourier transform between the pseudopolar and Cartesian grids. More importantly, it has been shown that an exact and faithful notion of the Radon transform can uniquely constructed along the lines of the pseudopolar grid. It is important to note that the history of such a grid dates back to 1974 to the work of Mersereau and Oppenheim who proposed a non-Cartesian "concentric square grid" in Fourier space for image reconstruction (Mersereau and Oppenheim, 1974). In the 1980's, such a grid was again utilized by Lawton *et al.* for tomographic reconstruction (Lawton, 1988 ; Edholm and Herman, 1987; Edholm *et al.*, 1988). However it was not until the recent work of Averbuch *et al.* that the mathematically faithful and algebraically exact PPFFT and its inverse was developed between the pseudopolar and Cartesian grids (Averbuch *et al.*, 2008a; Averbuch *et al.*, 2008b).

In contrast to the polar grid, the distance between sampling points along the lines of the pseudopolar grid varies from line to line. Subsequently, in order to map an equispaced object space input data onto a specific line on the pseudopolar grid, the fractional Fourier transform can be utilized to vary the output sampling distance of the Fourier transform. The 1D fractional Fourier transform (FrFT) is equivalent to the standard discrete Fourier transform with the exception of a factor of α (see Eq. 1), controlling the output spacing, in the exponent (the fractional Fourier transform reduces to the forward Fourier transform when $\alpha=1$ and to the inverse Fourier transform when $\alpha=-1$) (Bailey and Swartztrauber, 1991). Due to its similarity to FFT, the FrFT can be computed quickly using on order of $N \log(N)$ operations [17]. By choosing an appropriate value for α , the projection data can be mapped to the grid points of any line on the pseudopolar grid as shown in the next section. For the purpose of tomography, the FrFT of a projection can be related to the PPFFT of an object through an analog of the Fourier slice theorem which states that the 1D FrFT of a projection at an angle θ is equivalent to a slice of the 2D PPFFT of the image at the same angle θ in Fourier space (Averbuch *et al.*, 2008a; Averbuch *et al.*, 2008b).

2.2 Iterative Reconstruction Technique

While the PPFFT and its inverse provide an accurate method for the direct Fourier implementation of the Fourier slice theorem, experimentally an iterative algorithm is necessary to solve for the missing data and minimize the noise in order to properly address the limitations in the number of projections and source flux. Previously, an un-regularized iterative Fourier space algorithm, utilizing the pseudopolar transforms, was developed and experimentally implemented on the electron tomography modality (Miao *et al.*, 2005; Lee and Fahimian *et al.*, 2008); we build upon this method as follows.

As shown Figure 1(b), the method developed here is structured to reach a compromise between the constraints and experimental projections by iterating back and forth between object and Fourier space, enforcing constraints in object space and strict conformity with experimental projections in Fourier space. The algorithm utilizes oversampling, *i.e.* sampling the Fourier domain more finely than the Nyquist frequency, similar to the methodology used to solve the phase problem in lens-less diffraction microscopy (Miao *et al.*, 1999; Raines *et al.*, 2010). The effect of oversampling is to surround the object in real space by mathematical zeros, in a region called the support S , which the iterative algorithm can utilize as constraints. We note that the methodology can be experimentally implemented in two different ways. First, the projection data can be acquired at angles corresponding to the equally-sloped lines of the pseudopolar grid as first suggested in (Miao *et al.*, 2005) and as experimentally implemented in (Lee and Fahimian *et al.*, 2008); if so, the projections can be mapped onto the pseudopolar grid in Fourier space with machine accuracy using the FrFT, thereby eliminating all typical interpolations found in reconstruction algorithms. Second, as the PPFFT and inverse were first developed to provide an accurate substitute for the polar transform, conventional equally angled data can be mapped on to the pseudopolar grid accurately, as quantified by (Averbuch *et al.*, 2006), by rounding or interpolating to the nearest pseudopolar line. Furthermore, as inherently there exist a degree of inaccuracy in the alignment of projections at each angle experimentally, the said interpolation is justifiable. To emphasize this and the generality of the method, the experimental reconstructions presented here utilize conventional equally angled acquisitions.

The algorithm is initiated by first padding the projections with zeros and calculating the oversampled Fourier slices using the FrFT in a manner such that the output spacing matches the spacing of the pseudopolar grid at the corresponding pseudopolar line; mathematically this is given by

$$F_\theta(k)|_{k \leq R} = \sum_{r=1}^L p_\theta(r) \exp(-2\pi i \alpha r k / 2N) \quad (1)$$

where $F_\theta(k)|_{k \leq R}$ are the Fourier transform of the given projection $p_\theta(r)$ within the resolution circle, θ is the angle the projection makes with the isocenter, R is the radius of the resolution circle (as depicted in Figure 1(a)), α is equal to $1/\cos(\theta)$ for BH lines and $1/\sin(\theta)$ for BV lines, and L is an integer less than $2N$. The resolution circle exits because points beyond it cannot be represented by the Fourier transform of the experimental data and must be solved for by the iterative algorithm (Miao *et al.*, 2005).

The j th iteration of the iterative algorithm can be broken down into the following steps: (i) Application of the inverse PPFIT to the Fourier space data $F_\theta^j(\mathbf{k})$ to obtain an object space image $f^j(\mathbf{r})$; if this is the initiating iteration, the Fourier data is the aggregation of the transformed experimental projections with the regions corresponding to the missing projections set to 0 or random numbers; (ii) Derivation of a new object through the regularization $f^j(\mathbf{r})$ over a domain Ω . This step provides a flexible structure that can incorporate any regularization constraints such as existing variational methods which have been widely studied for the purpose of denoising. These methods treat image as a function in certain function space and evolve the image under nonlinear partial differential equations or the corresponding minimization problems. Most notably, total variation based optimization have garnered attention for edge preserving regularization (Rudin *et al.*, 1992) as well as in the realm of compressed sensing (Candes *et al.*, 2006). However, the use of pure total variation models for realistic images have been shown to produce artificial patches; this is due to the choice of the bounded-variation space and the corresponding total variation norm. More recently, the nonlocal means regularization model has been introduced which denoise a pixel by averaging the nearby pixel values with similar patterns. The basic assumption behind is that a natural image contains repeating structures instead of repeating pixels. This method has been proved to be successful to remove artifacts while keeping the regular pattern and texture containing in the image and has been extended to include variational method using functionals with nonlocal regularization (Gilboa and Osher, 2008), and proven superior to many other image regularization methods as it considers the large-scale structure of the image beside the local differences between pixels, which makes it capable of preserving important detailed features in an image while removing artifacts effectively (Buades *et al.*, 2005). For these reasons, in this paper we present the first incorporation of total variational implementation of the nonlocal means (NL-means) algorithm for regularized tomographic reconstruction. The NL-means algorithm is operated on $f^j(\mathbf{r})$ to yield $\bar{f}^j(\mathbf{r})$, which is defined as the solution of the following optimization problem and can be obtained by steepest descent method or other recent advanced method:

$$\bar{f}^j = \arg \min_u J_w(u) + \frac{\lambda}{2} \|u - f^j\|^2 \quad (2)$$

where

$$J_w(u) = \int_{\Omega} \sqrt{\int_{\Omega} (u(x) - u(y))^2 w(x, y) dy} dx \quad (3)$$

is the nonlocal total variation with respect to the weight function describing the similarity between the patches around different pixels. Given reference image u and a filter parameter h , $w_{u,h}$ is a function on $\Omega \times \Omega$, defined as

$$w_{u,h}(x, y) = \exp \left\{ - \frac{G * (\|u(x+\cdot) - u(y+\cdot)\|^2)(0)}{2h^2} \right\} \quad (4)$$

where

$$G * (\|u(x + \cdot) - u(y + \cdot)\|^2)(0) = \int_{\mathbb{R}^2} G(t) \|u(x + t) - u(y + t)\|^2 dt \quad (5)$$

and G is the Gaussian kernel.

(iii) Application of a positivity constraint by setting

$$f'^j(\mathbf{r}) = \begin{cases} \bar{f}^j(\mathbf{r}) & \text{if } \mathbf{r} \notin S \text{ and } \bar{f}^j(\mathbf{r}) \geq 0 \\ 0 & \text{if } \mathbf{r} \in S \text{ or } \bar{f}^j(\mathbf{r}) < 0 \end{cases} \quad (6)$$

where S represents the support region due to the oversampling that is expected to contain zeros; (iv)

Calculation of the new Fourier space data $F'^j(\mathbf{k})$ through the application of the PPFST to $f'_j(\mathbf{r})$; (v)

Updating the corresponding experimentally measured Fourier slices, $F_\theta(\mathbf{k})|_{|\mathbf{k}| \leq R}$, and retaining the Fourier slices outside the resolution circle, $F'^j_\phi(\mathbf{k})|_{|\mathbf{k}| > R}$ and the missing Fourier slices, $F'^j_\phi(\mathbf{k})$ by

$$F^j_\varphi(\mathbf{k}) = F_\theta(\mathbf{k})|_{|\mathbf{k}| \leq R} \cup F'^j_\phi(\mathbf{k})|_{|\mathbf{k}| > R} \cup F'^j_\phi(\mathbf{k}) \quad (7)$$

where $\varphi = \theta \cup \phi$ is the complete set of angles for the pseudopolar grid, θ is the set of angles of the measured projections, and ϕ is the set of angles of the missing projections. In contrast to the 3D algorithm presented in (Miao *et al.*, 2005), this algorithm does not contain a FFT across 2D slices in step (iv) nor it's inverse FFT in step (i); as a result, this 2D form allows the algorithm to be readily parallelized by sending each 2D slice to a different CPU for 3D reconstructions. During each iteration, an error function defined by

$$Error = \frac{\sum_{\mathbf{k}, \theta} |F^j_\theta(\mathbf{k})|_{|\mathbf{k}| \leq R} - F_\theta(\mathbf{k})|_{|\mathbf{k}| \leq R}|}{\sum_{\mathbf{k}, \theta} |F^j_\theta(\mathbf{k})|_{|\mathbf{k}| \leq R} + F_\theta(\mathbf{k})|_{|\mathbf{k}| \leq R}|} \quad (8)$$

is calculated. The iterations are automatically terminated when the error does not fall by certain percentage from previous iterations. In all simulations performed here, the algorithm was stopped when the error did not decrease by more than 1% from the tenth previous iteration. In order to be completely consistent with the experimental data, the output of the algorithm is always the image resulting from the inversion of the Fourier data once the data has been updated with the experimental Fourier slices; no constraints or processing is applied after this point. The computation time of each iteration is on the order of a single FBP reconstruction; with respect to this, recently we have developed a gradient descent version of the EST algorithm which reduces the computation time for each iteration by up to 70% (Mao *et al.*, 2009); this algorithm is beyond the scope of this paper, but it will be explored in subsequent papers.

3. Methods

3.1 Experimental and Simulation Methods

To quantify the performance of the algorithm and evaluate its utility for limited angle tomography, tomographic reconstructions were performed as function of the number of projections using both simulated and experimental data in the x-ray absorption and phase contrast CT modalities. The interaction of x-rays for these modalities can be quantified by the complex refractive index

$$n = 1 - \delta - i\beta \quad (9)$$

where the projection data in phase-contrast imaging is given by the phase change after transmittance of x-rays of wavelength λ through a distance of s

$$\Phi = \frac{2\pi}{\lambda} \int \delta(s) ds \quad (10)$$

while the projection data for absorption imaging is given by

$$-\log\left(\frac{I}{I_0}\right) = \frac{4\pi}{\lambda} \int \beta(s) ds \quad (11)$$

where I_0 and I are the un-attenuated and attenuated intensities, respectively.

Simulations were used to model the possible application of tunable monochromatic compact synchrotron x-ray absorption CT for biomedical applications. For this purpose, a segmented anthropomorphic phantom based upon the scans of a human by Zubal, *et al.*, was constructed at a resolution of 3.24 mm (Zubal *et al.*, 1994). A monochromatic source with energy of 50 KeV was used, as this would be the approximate dosimetric optimal energy based on Kerma considerations for a biomedical sample containing bone (Lewis, 2004); using mass attenuation coefficients and densities from the NIST and ICRU data sets, corresponding linear attenuation coefficients were assigned to the segmented tissues (ICRU, 1989). To more rigorously quantify the image quality, three sets of resolution markers of varying contrast were added to interior of skull as shown in Figure 1(c); these consisted of squares of 1x1 pixel separated by 1 pixel, squares of 2x2 pixels separated by 2 pixels, and squares 3x3 pixels separated by 3 pixels; since the resolution may have dependencies on contrast, 3 sets of materials was used for each marker set, consisting of high contrast bone, medium-low contrast tissue with linear attenuation coefficient 30% above surrounding brain mater, and air were assigned to the arrays (linear attenuation coefficients of 81.4 m^{-1} , 40.7 m^{-1} , and 0.0 m^{-1} , respectively). As with most synchrotron sources, the simulated source to detector geometry was parallel. The number of simulated detector elements at each angle was 256 for which tomographic projections of the phantoms were calculated using an accurate Radon transform, based on trigonometric interpolation, along a set of equally-angled and equally-sloped lines for the conventional and EST reconstructions, respectively. The projections at each detector element position were exponentiated and weighted by an un-attenuated flux, I_0 , to give the noiseless intensity at detector. Poisson noise, as approximated by the Gaussian distribution with a standard deviation of for counts greater than 30, was added to the intensities; the noisy intensities were subsequently inverted to arrive at the noisy projections. Based on the signal to noise ratio of experimental projection data of current implementation of x-ray absorption CT was estimated, the un-attenuated flux I_0 was approximated to $5.0 \times 10^5 \text{ m}^{-2}$ for the simulations.

Experimentally, an image quality phantom consisting of three material inserts, a high intensity filament, and a micro beads resolution insert, was scanned at ESRF to acquire both x-ray absorption and phase contrast projections. Specifically, the body of the phantom consisted of a 7.5 mm polymethylmethacrylate (PMMA) cylinder, and the three material inserts consisted of de-mineralized water, 99.6% ethanol, a lipid, the 0.125 mm filament consisted of 99.99% pure aluminum; the beads in the resolution insert consisted of a mixture of 0.1 mm and 0.2 mm monosized polymers. The x-ray source was produced at the 150 m imaging beamline at the ID19 experimental station at ESRF. For the x-ray absorption experiment, the beam energy was 15 keV, the source to phantom distance was 32 mm, and the effective detector size after binning to 512 detector elements was 29.8 μm . A very fast acquisition method was employed to acquire the absorption data for which 200 projections were acquired uniformly in 3.6 s (1.8 ms/projection) by rotating the phantom. For the phase contrast experiments, a beam energy of 24 keV was used, and the phase data was attained using the in-line acquisition method coupled by the mixed phase retrieval algorithm presented in (Langer *et al.*, 2008); the phantom to detector distance for the in-line method were 0.012, 0.1, 0.3, and 0.99 m, for which 1200 radiographs uniformly about 180° were detected at each distance; the detector was a FReLon camera with an effective element size of 30 μm after binning.

For reference, all data was also reconstructed using FBP reconstructions, for which the projections were padded with zeros, and the un-cropped Ram-Lak (*i.e.* ramp filter) in conjunction with shape-preserving piecewise cubic interpolation for the backprojection process was utilized (Kak and Slaney, 2001); since it was shown that the previous EST algorithm resulted in higher resolution images than FBP (Miao *et al.*, 2005; Lee and Fahimian *et al.*, 2008), the un-cropped Ram-Lak filter was specifically selected in order to avoid degradation of resolution due the suppression high frequency data which may result through the use of other filters (Comtat *et al.*, 1998; Kak and Slaney, 2001; Natterer, 2001). The equally-sloped projection data of the simulations was reconstructed using the algorithm developed in section 2.2. As mentioned, the pseudopolar transform also provides an accurate approximation to the polar Fourier transform, and consequently, conventional data sets acquired at equal angle intervals can also be utilized by the algorithm developed here. To demonstrate this, and as well as to isolate the effect of the developed algorithm from the problem of interpolation, experimentally the same equally angled projections were utilized for both the EST and FBP reconstructions; although this removes the interpolation accuracy advantage of equal slope projections, it serves to quantify the effect of the algorithm, which is the subject of this work. Subsequently, for the EST reconstructions, the polar projections were interpolated to the nearest pseudopolar line as detailed in section 2.2.

3.2 Methods for Quantification of Image Quality

As no one metric can adequately quantify the image quality of a reconstruction, an aggregate of metrics are calculated in this paper as follows. For simulations, where the original phantom is known, correlation to the original can be used as one measure of the accuracy of the reconstructions. A rigorous method to measure the correlation of complex images is by comparing Fourier coefficients as a function of spatial frequency through the Fourier Ring Correlation (FRC) (van Heel M and M, 2005; Frank, 2006). The FRC is capable of quantifying the resolving power as it provides a generalization of the MTF (Nickoloff, 1988; Nickoloff and Riley, 1985) for complex objects, and reduces to the MTF when the comparison and reconstructed image are an impulse and point spread function. The FRC across ‘rings’ in Fourier space is defined as

$$FRC_{rec,phant}(k, \Delta k) = \frac{\left| \sum_{\mathbf{k}} F_{rec}(\mathbf{k}) F_{phant}^*(\mathbf{k}) \right|}{\sqrt{\sum_{\mathbf{k}} |F_{rec}(\mathbf{k})|^2 \sum_{\mathbf{k}} |F_{rec}(\mathbf{k})|^2}} \Bigg|_{\mathbf{k} \in [k, k + \Delta k]} \quad (12)$$

where k represents the radial parameter in the Fourier space, Δk represents the frequency interval size of the rings such that $k \in [k, k + \Delta k)$, and the subscripts *rec* and *phant* refer to the reconstructed image and the original phantom, respectively. A FRC value of 1 represents 100% correlation while a value of 0 represents 0% correlation; the spatial frequency defined by the 0.5 value of the FRC curve, representing 50% correlation between reconstructed image and the original phantom, is commonly taken as a numerical value of the resolution.

In order to quantify the resolution, and its dependency on contrast, a series of resolution markers have been added to the interior of the anthropomorphic phantom. Experimentally, one insert of the PMMA phantom includes aggregates of small polymer beads, for which the separating space between the beads can be used to evaluate the resolution. To measure the noise response of the two different techniques, the SNR, defined by the mean intensity of a region divided by the standard deviation, was measured for each segmented tissue region of the Zubal phantom and regions of interest in the experimental data. Additionally, the CNR, defined here as the absolute value of the means of two segment regions divided by the mean of the standard deviations of the two regions, was used to quantify the contrast between tissues

of interest and experimental material inserts.

4. Results

4.1 Simulations

Simulations on slice 82 of the modified Zubal anthropomorphic head phantom (Figure 1(c)-(d)) were carried out to quantify the image quality of the method as a function of the number of projections and its performance relative to standard equally-angled FBP. Poisson noise was added to the projections corresponding to a fluence of $5.0 \times 10^5 \text{ m}^{-2}$. As the head is located within the central 160×160 pixel portion of the field of view, according to the Nyquist radial sampling criteria, the number projections required to avoid streaking artifacts for a pixel phantom is approximately $160\sqrt{2}(\pi/2) \sim 360$ projections (Kak and Slaney, 2001). Subsequently, a series of simulations were performed by starting at a maximum of 360 projections and then systematically reducing the number of projections. For the FBP reconstructions, all projections were calculated at equal-angle increments uniformly distributed across 180° , while for EST reconstructions, the projections were calculated along equally-sloped lines of the pseudopolar grid distributed across 180° at constant slope increments. The results for reconstructions with under-sampled number of projections of 60, 90 projections, as well as the Nyquist sampled 360 projections are shown in Figure 3. For the EST reconstructions, the algorithm was terminated automatically when the previously mentioned error condition was reached, resulting in 65, 45, and 19 iterations for the reconstructions of 60, 90, and 360 projections respectively. Figure 3 quantifies the resolution and quality of the reconstructed images via the FRC with the original phantom, while Tables 1 and 2 quantify the image quality via measurements of the SNR and the CNR for tissues of interest.

For the same number of projections, the FRC curves for the EST reconstructions are significantly higher than the corresponding FBP reconstructions among all spatial frequencies indicating higher resolution and correlation for the EST reconstructions; the results indicate that the EST reconstructions with a reduced number of projections of 90 have higher correlation with the original phantom across all spatial frequencies than the fully sampled FBP with 360 projections. The visual comparison with the original phantom in Figure 1(c)-(d) with reconstructions in Figure 3 indicate sharper boundaries and visual correlation for the EST reconstructions. Alternative to the FRC, the resolution is visually quantified by resolution markers; it is noted that medium contrast resolution markers are not well delineated in the FBP reconstruction. The results in Table 1 and 2 demonstrate that, for the same flux, the EST reconstructions results in an average SNR that is 3.2 to 3.7 greater than the corresponding FBP reconstructions, and in an average CNR that is 2.6 to 4.2 greater than the FBP reconstructions, for the same number of projections; also it is noted that for nearly all tissues, the EST reconstruction with a reduced number of projection result in a higher SNR and CNR than the FBP reconstruction at the full Nyquist sampling. The SNR and CNR results are visually confirmed in the greater recovery of low contrast objects (such as in the mid-brain region), elimination of streaking artifacts, and reduction of overall statistical noise.

4.2 Experimental Results

4.2.1 X-ray Absorption Microtomography

The experimental results for the x-ray absorption modality are displayed in Figure 4; from left to right, the first column are the various regions of interest for the full dose FBP reconstruction with 200 projections, the 60% dose reduced FBP reconstruction with 80 projections, and the 60% dose reduced EST reconstruction with 80 projections, respectively; from top to bottom, whole phantom view, medium

contrast region, low contrast region, and high contrast resolution region. Visually, it is observed that the dose reduced FBP reconstruction in the second column is degraded by streaking artifacts and noise caused by the under-sampling of the projections while the dose reduced EST reconstruction has recovered the missing data. To quantify reconstruction results, SNR and CNR were calculated for regions of interest; the SNR was defined as the mean pixel intensity of a region divided by the standard deviation of the intensities; the CNR between two regions of interest was defined as the absolute value of the difference of the mean of the first region and the second region divided by the mean of the standard deviations of the first region and the second region.

The results indicate that there is a significant deterioration in both SNR and CNR going from 200 projections to 80 projections for the FBP reconstructions; on average, both the SNR and CNR are approximately 50% lower for the FBP reconstruction with 80 projections than the FBP projection with 200 projections. On the other hand, the dose reduced EST reconstruction with 80 projections outperforms the dose reduced FBP reconstruction, but also outperforms the full dose FBP reconstruction among all measured metrics of SNR and CNR. On average, relative to the FBP reconstruction with the same number of projections, the SNR for the EST reconstruction was 3.5 times greater, and the CNR is 3.4 times greater; relative to the full dose FBP reconstruction, the SNR for the 60% dose reduced EST reconstruction is 1.8 times greater, and the CNR is 1.7 times greater. Visual comparison of the beads and the visibility of small void regions in between them of the high contrast resolution region suggest that the EST reconstruction has equal or better resolution than both FBP reconstructions.

4.2.2 X-ray Phase Contrast CT

Experimental results for the x-ray phase contrast CT modality are shown in Figure 4. As with x-ray absorption experiments, the same equally angled data was utilized for the FBP and EST reconstructions. The first column represents FBP with 200 projections which is below the Nyquist requirement, but represents an image quality that is acceptable for conventional reconstructions. The second and third columns represent the FBP and EST reconstructions with 60 projections, respectively. From top to bottom, comparison of the full field of view, contrast insert, resolution insert, and zoomed area of resolution insert. In general, it is noted that streaking artifacts degrade the under-sampled FBP reconstruction while the EST reconstructions are void of such artifacts. To quantify the image quality, the SNR and CNR of the material insert shown was calculated relative to the PMMA background. The SNR was measured to be 32.1, 10.6, and 71.6 for the 200 projection FBP, 60 projection FBP, and 60 projection EST reconstruction respectively. The CNR was measured to be 9.7, 3.4, and 19.9 for the 200 projection FBP, 60 projection FBP, and 60 projection EST reconstruction respectively. The results are consistent with the visual appearance of second row which suggests that the 60 projections EST reconstruction is smoother than that of both the 60 and 200 projections FBP reconstruction. The resolution is visually assessed in the third and fourth row. The 60 projection FBP has visibly noticeable lower resolution as determined by the separation of the beads when compared to the 200 projection FBP, while the 60 EST reconstruction presents with comparable separation of fine features to the 200 projection FBP.

5. Conclusion

Tomographic reconstruction from a limited number of projections provides a method to simultaneously reducing the radiation dose, and in many cases the acquisition time. We have developed a method of tomographic acquisition and regularized iterative Fourier-based reconstruction, termed Equally-Sloped Tomography (EST), which through the use of mathematical and physical constraints, provides a methodology for tomographic reconstruction using only a fraction of the number of projections as

conventionally required in tomographic application. Simulation and experimental results in both the x-ray absorption and phase contrast CT modalities have indicated feasibility of reducing the number of projections by a factor of 60-75% while still achieving equal or better image quality relative to conventional reconstructions. The combination of the developed methodology and synchrotron x-ray sources hence allows for low dose x-ray absorption and phase-contrast CT applications in biology and medicine. Finally, as methodology pertains to general tomographic reconstruction, it can also be applied to medical x-ray CT scanners for low-dose image reconstructions, which will be presented in subsequent papers.

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FIGURE CAPTIONS

Figure 1 Methodology: (a) Graphical relationship between the oversampled pseudopolar grid and the corresponding Cartesian grid, where $N = 8$. The black and white points delineate the Basically Horizontal (BH) and Basically Vertical (BV) groups, respectively, and the dashed circle represents the resolution circle. (b) The j^{th} iteration of the iterative algorithm as described in section 2.2; dashed line indicates that the step occurs outside the loop in the initiating or terminating step. (c) Slice 82 of the Zubal head phantom with assigned linear attenuation coefficients (Zubal *et al.*, 1994). (d) Zoomed view of critical structures for comparisons with results of Figure 3.

Figure 2 FBP and EST reconstructions of the Zubal head phantom. FBP reconstructions with 60, 90, and 360 projections are presented in (a), (c), (e), respectively, with the corresponding EST reconstructions presented in (b), (d), (f).

Figure 3 Fourier ring correlation of the reconstructed images with the original phantom for results presented in Figure 2.

Figure 4 Experimental results for the x-ray absorption CT modality. From left to right, the columns represent FBP with 200 projections, FBP with 80 projections, and EST with 80 projections. From top to bottom, comparison of the full field of view, medium contrast insert, low contrast insert, and resolution insert.

Figure 5 Experimental results for the x-ray phase contrast CT modality. From left to right, the columns represent FBP with 200 projections, FBP with 60 projections, and EST with 60 projections. From top to bottom, comparison of the full field of view, contrast insert, resolution insert, and zoomed area of resolution insert.

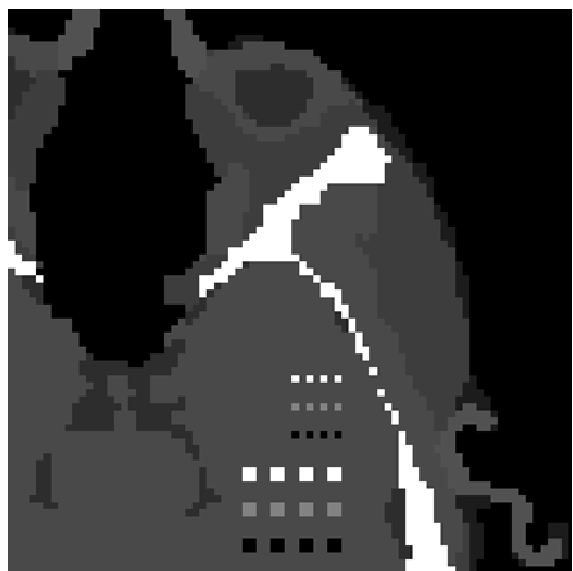
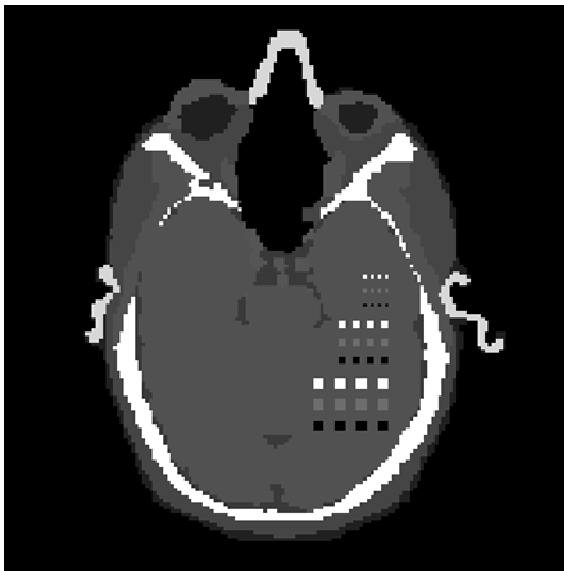
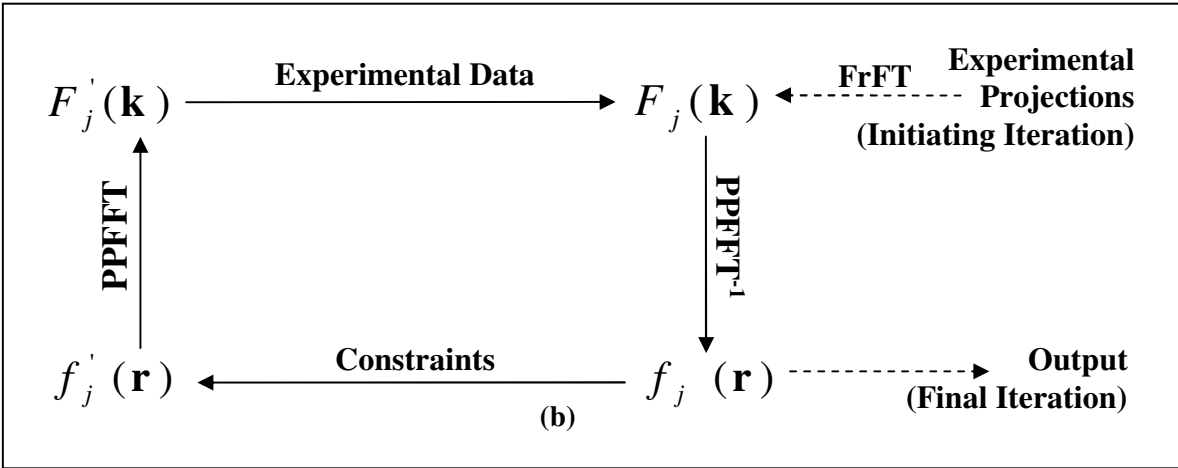
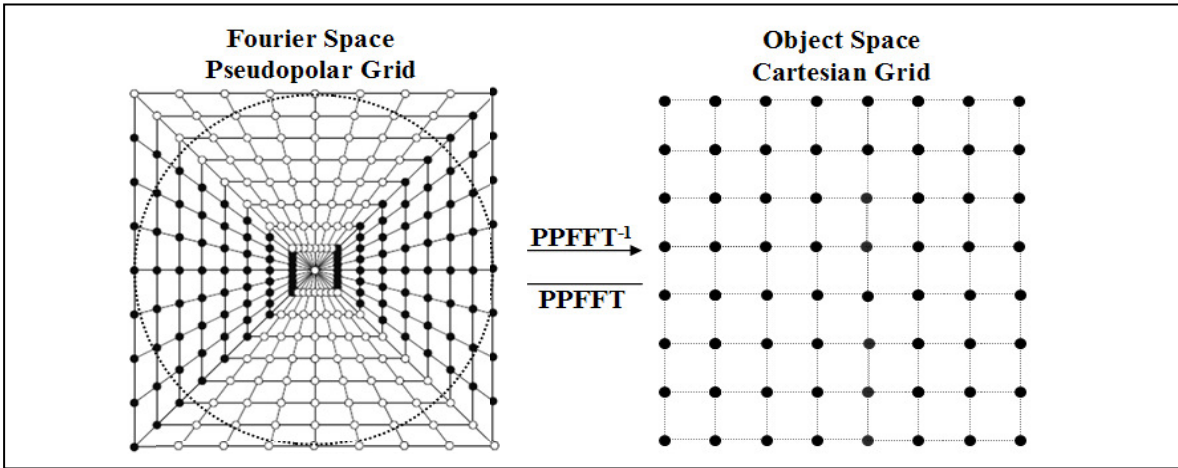


Figure 1

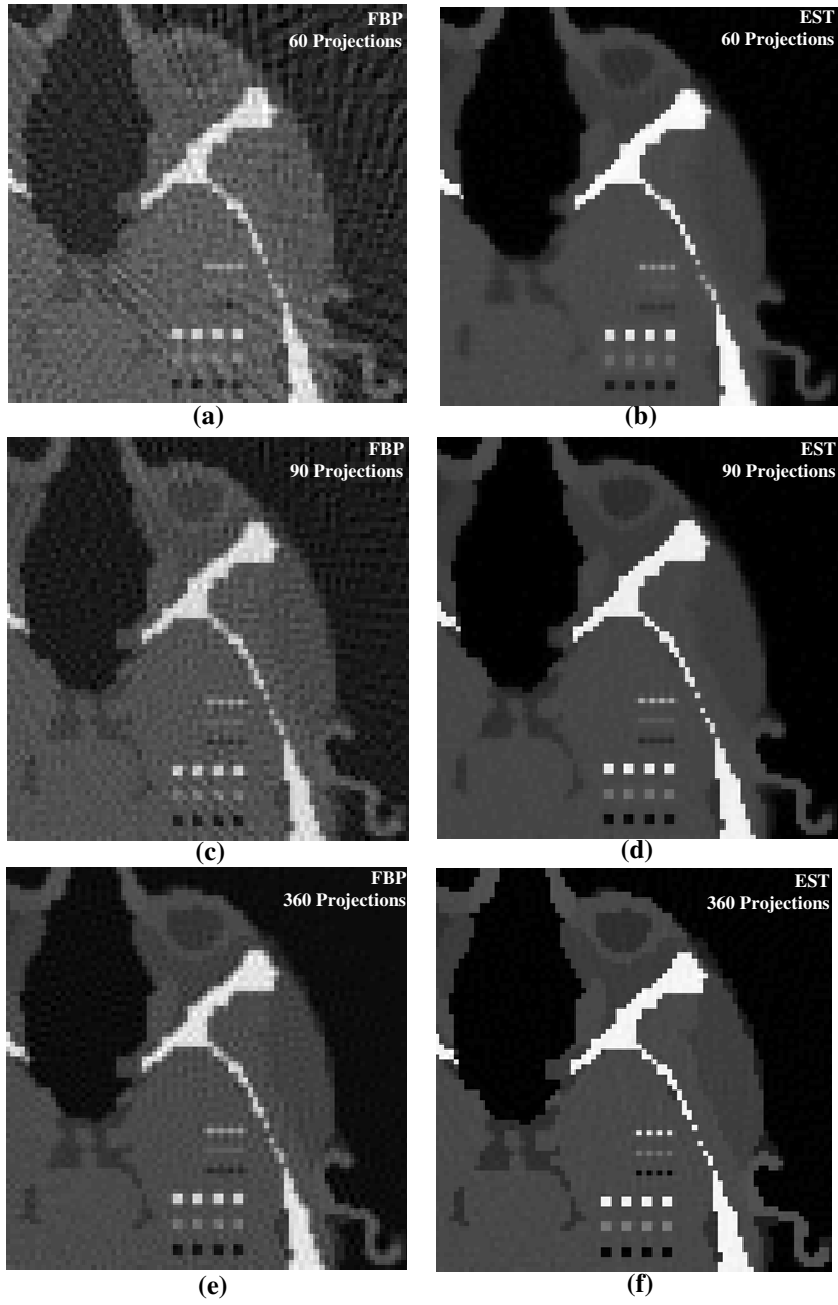


Figure 2

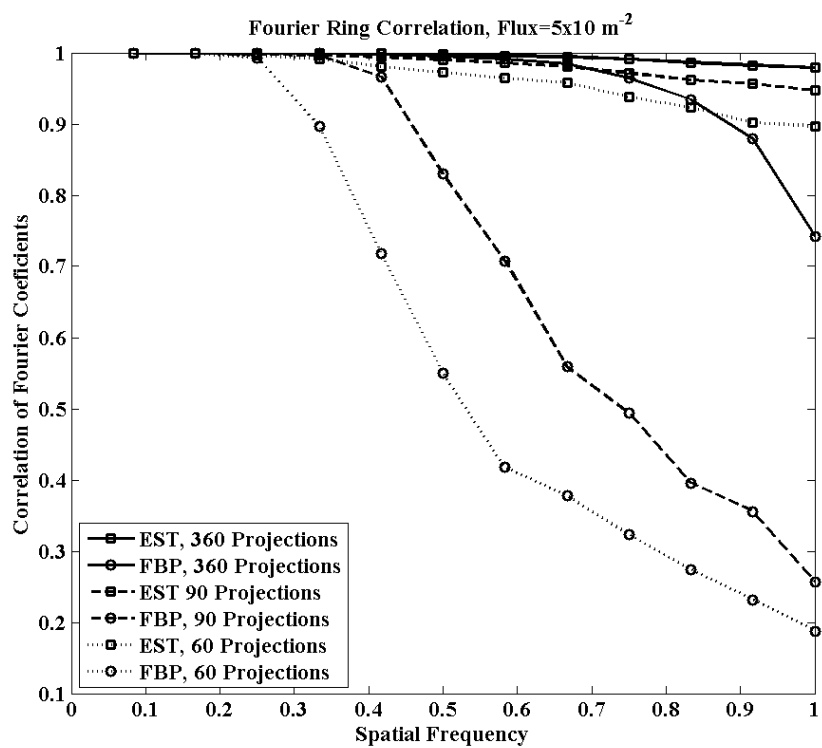


Figure 3

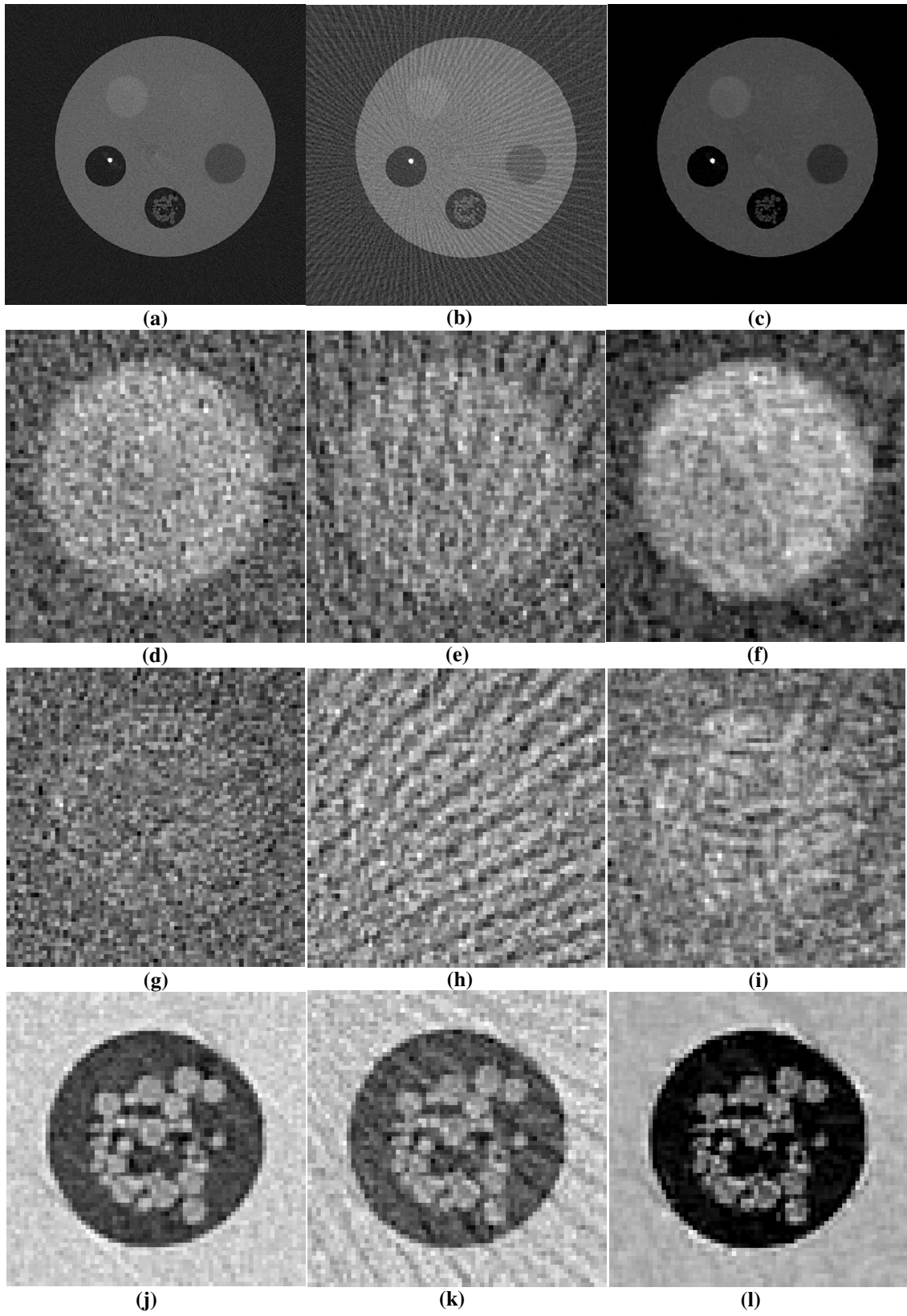


Figure 4

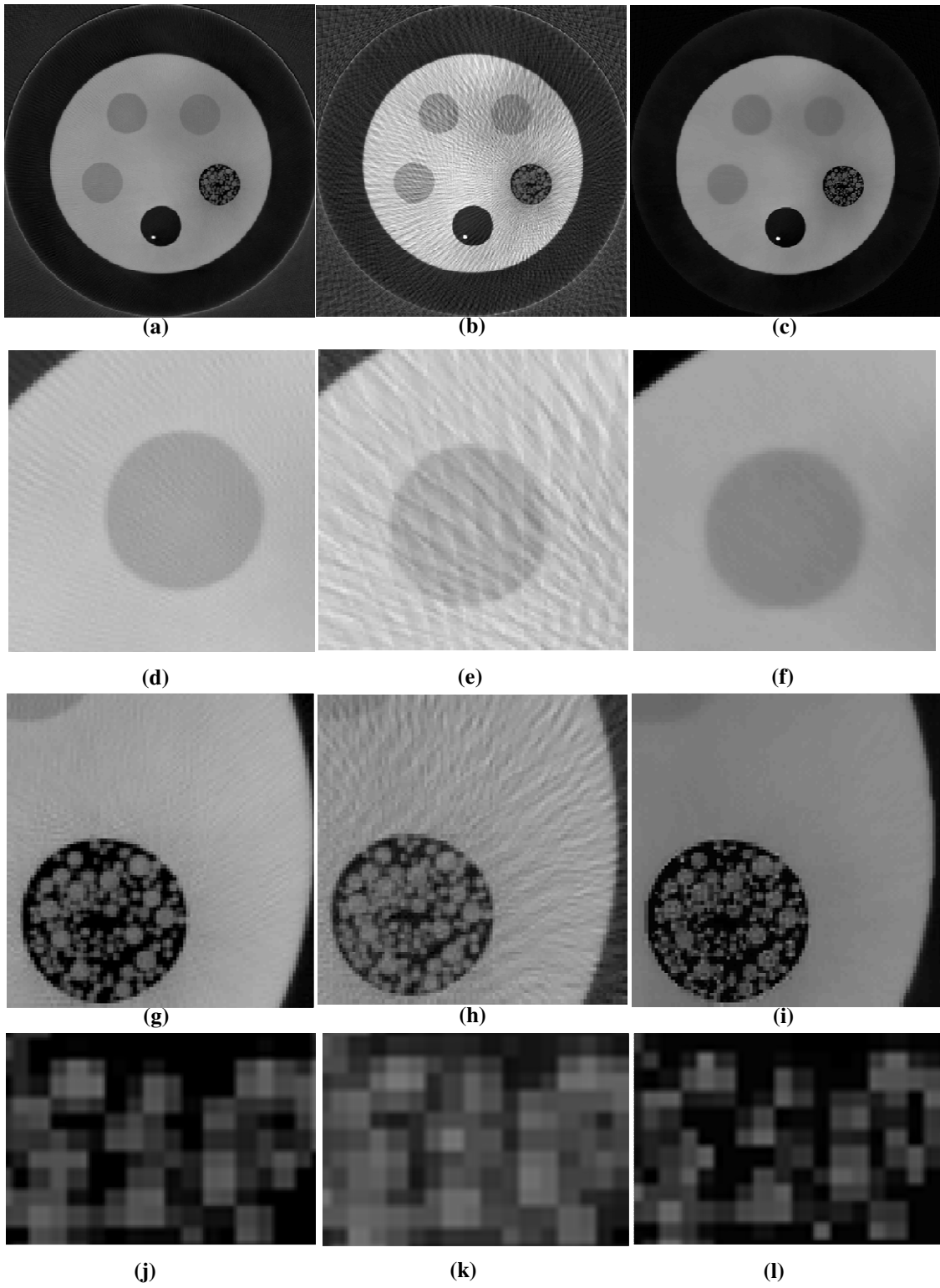


Figure 5

Table 1: Comparison of Signal to Noise Ratio for Tissues of Interest

Projections Flux=$5 \times 10^5 \text{ m}^{-2}$	Method	Bone	Cartilage	Brain	Muscle	Fat	Cerebral Fluid	Eye Ext.	Eye Int.	Skin	Mean of EST/FBP
60	FBP	8.5	3.9	4.3	3.6	3.1	2.4	3.7	2.6	1.6	
60	EST	17.7	9.2	21.7	10.5	11.6	6.5	14.3	13.2	3.2	3.2
90	FBP	10.1	5.7	6.2	4.6	4.1	3.1	6.2	4.9	2.3	
90	EST	29.9	15.5	27.5	16.6	14.7	8.8	18.1	14.6	4.0	3.2
360	FBP	11.2	8.8	9.8	5.6	5.5	3.7	12.8	11.0	4.1	
360	EST	64.6	36.2	32.2	27.5	26.4	16.1	32.1	24.0	10.7	3.7

Table 2: Comparison of Contrast to Noise Ratio for Tissues of Interest

Projections Flux=$5 \times 10^5 \text{ m}^{-2}$	Method	Brain to C. Fluid	Muscle to Fat	Bone to Muscle	Bone to Cartilage	Eye Ext. to Eye Int.	Skin to Fat	Skin to Air	Mean of EST/FBP
60	FBP	1.1	0.6	6.8	6.9	1.3	1.7	1.6	
60	EST	3.1	1.8	16.4	16.1	4.3	4.4	4.8	2.6
90	FBP	1.5	0.8	8.2	8.9	2.3	2.4	2.4	
90	EST	4.8	2.5	27.5	26.4	5.7	5.5	6.1	3.0
360	FBP	2.0	1.0	9.4	11.0	4.8	3.7	5.9	
360	EST	9.6	4.4	54.0	56.1	11.9	12.5	14.6	4.3

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