FAST EDGE-FILTERED IMAGE UPSAMPLING

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ABSTRACT

We propose a novel piecewise hyperbolic operator for rapid upsampling of natural images. The operator uses a slope-limiter scheme that conveniently lends itself to higher-order approximations and is responsible for restricting spatial oscillations arising due to the edges and sharp details in the image. As a consequence the upsampled image not only exhibits enhanced edges, and discontinuities across boundaries, but also preserves smoothly varying features in images. Experimental results show an improvement in the PSNR compared to typical cubic, and spline-based interpolation approaches.

***Index Terms*—** interpolation, edge-preserving, slope-limiter

1 Introduction

Interpolation is a frequently needed tool for many imaging applications ranging from image zooming, resizing, retouching, formatting, manipulation, and compositing. Aside from applications aiming to aesthetically manipulate images, one routinely needs image resampling for the purpose of image matching and registration for computer vision applications. Often times, high throughput machine vision tasks such as scene reconstruction and warping utilize simple but fast techniques such as bilinear, bicubic, and occasionally b-spline interpolation for single images. These methods implicitly assume a smoothness prior in the image resampling process. For example, bilinear interpolation assumes that the resampled intensity value arises from first order local averaging of neighboring intensities of the image, whereas higher order methods such as bicubic, and b-spline assume that local intensities are estimated by imposing smoothness constraints by fitting high-order polynomials to the intensity function of the image. While bilinear interpolation restricts signal overshoots at discontinuities, bicubic, b-spline and other higher order methods introduce ringing, and haloing artifacts in images. Furthermore, the performance of many vision applications rely on accurate preservation and detection of edges from images. Thus a compromise needs to be achieved between edge fidelity and processing latency.

There are several approaches for image upsampling, especially for preserving edges [1, 2, 3] during the interpolation process. Our approach focuses on single image upsampling, and is different from image super-resolution approaches [4] that typically involve either fusion of multiple images, or integration of example-based constraints along with sub-pixel homologies. Conventionally, the problem of image upsampling is approached by initially formulating a degradation model specified by a convolution kernel as well as a downsampling operator. The high resolution image is then reconstructed by solving the inverse problem of image reconstruction that assigns intensity values to the desired image. In this paper, instead of explicitly optimizing over the degradation model, we directly focus on the upsampling operator that yields a one-step interpolation of the observed image. Our resampling approach borrows from methods in fluid dynamics [5, 6] that attempt to construct total variation diminishing (TVD) solutions from high resolution numerical schemes for modeling shocks and discontinuities in fluid flows. Our interpolation method does not require iterative optimization; instead it provides a one-step up/downsampling of the original image. It relies on higher order derivatives of the image, and limits spatial oscillations at edges and discontinuities, and at the same time, satisfies the dual requirement of speed and edge accuracy.

2 Edge-Preserved Sampling

This section formulates the upsampling problem, and introduces the edge-preserving operator based on a slope-limiter function. The single frame degradation model is usually expressed as a convolution of the high resolution image with a blur kernel operator, followed by a down-sampling process. Thus a low resolution image f is written as $f = D(h*u)+\eta$, where D is a downsampling operator, and η is zero-mean Gaussian noise. Previously Marquina et al. [7] have proposed a TV regularization solution for high resolution image reconstruction given by $\hat{u} = \text{argmin}_u\{TV(u) + \frac{\lambda}{2}[||f - D(h * u)||^2_{\mathbb{L}^2} - \sigma^2]$. They iteratively construct a high resolution im-

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age by solving the Euler Lagrange equation under homogeneous Neumann boundary conditions. In this paper, instead of solving the regularization problem, we are only interested in seeking a minimum for the error condition

$$E = ||u - SD(h * u)||^2, \quad (1)$$

where S is an upsampling operator, and $D \circ S = I$, an identity matrix. Here, the operation of downsampling followed by upsampling is not reversible, and thus $D \circ S \neq S \circ D$. The observed image f is confounded by both the down sampling process, and the blur operator. Assuming that the operators D and h, are fixed, and independent of the image u in Eq. 1, there are a variety of upsampling operators S, that seek to minimize the cost in Eq. 1. For example, it can be shown that for piecewise smooth images, a bilinear interpolant operator yields a lower estimate for the error E in equation 1 when compared with a nearest neighbor interpolant. In this paper, instead of proposing a variational formulation to solve Eq. 1, our goal is to seek specific upsampling operators S that preserve or enhance edges in the image.

2.1 Higher-order Piecewise Hyperbolic Operator

In order to fix notation, we consider a two-dimensional $m \times n$ image, and set up a pixel $u_{jk} : 1 \leq j \leq m, 1 \leq k \leq n$, where u_{jk} is an average of the true signal intensity $g(x, y)$ in the pixel, and is written as

$$u_{jk} = \frac{1}{h_x h_y} \int_{x_j - h_x/2}^{x_j + h_x/2} \int_{y_k - h_y/2}^{y_k + h_y/2} g(x, y) dx dy, \quad (2)$$

where h_x and h_y are the pixel step sizes along the X and Y dimension of the image. Furthermore, we assume that the domain of the pixel function u_{jk}, centered at (x_j, y_k) is given by $(x_j - \frac{h_x}{2}, x_j + \frac{h_x}{2}) \times (y_k - \frac{h_y}{2}, y_k + \frac{h_y}{2})$, and denote the divided differences in the image by $\Delta_+^x u_{jk} = \frac{u_{j+1k} - u_{jk}}{h_x}$, $\Delta_-^x u_{jk} = \frac{u_{jk} - u_{j-1k}}{h_x}$, and $\Delta_+^y u_{jk} = \frac{u_{jk+1} - u_{jk}}{h_y}$, and $\Delta_-^y u_{jk} = \frac{u_{jk} - u_{jk-1}}{h_y}$.

Our goal is to approximate the function $g(x, y)$ in each pixel by means of an elementary function $H_{jk}(x, y)$, such that Eq. 2 is satisfied. While there are several different edge-preserving forms for the function H_{jk}, following Marquina [6] we restrict our focus to special type of functions also known as slope-limiters. The idea here is to choose a slope-limiter form such that the interpolant ensures accurate spatial approximation, and prevents excessive increase in the total variation in the pixel u_{jk}. There is a wide variety of such nonlinear functions [8] that preprocess divided differences and enforce the order of accuracy. In this paper, we propose a third order approximation to the function $H(x, y)$ using a piecewise hyperbolic form. Additionally we will use the harmod limiter function [6] that uses the notion of a harmonic mean instead of an absolute mean of the divided differences. The harmod limiter operator is given by $\text{harmod}(\alpha, \beta) = \frac{\text{sgn}(\alpha) + \text{sgn}(\beta)}{2} \cdot \frac{2\alpha\beta}{\alpha + \beta}$. The interpolant function can then be defined as

$$H_{jk} = u_{jk} + \frac{a_{jk}}{x - x_j + c_{jk}} + \frac{b_{jk}}{y - y_k + d_{jk}}, \quad (3)$$

where the parameters a_{jk}, b_{jk} are given by $a_{jk} = \text{harmod}\big(\frac{u_{jk} - u_{j-1k}}{h_x}, \frac{u_{j+1k} - u_{jk}}{h_x}\big)$, and $b_{jk} = \text{harmod}\left(\frac{u_{jk} - u_{jk-1}}{h_y}, \frac{u_{jk+1} - u_{jk}}{h_y}\right)$. In order to represent a function H of the form given in Eq. 3, we consider the following ansatz [6]

$$H_{jk}(x, y) = u_{jk} + a_{jk} \frac{h_x}{\alpha_x^2} \big[\log(\frac{2 - \alpha_x}{2 + \alpha_x}) - \frac{h_x}{x - x_j - \frac{h_x}{\alpha_x}} \big] + b_{jk} \frac{h_y}{\alpha_y^2} \big[\log(\frac{2 - \alpha_y}{2 + \alpha_y}) - \frac{h_y}{y - y_j - \frac{h_y}{\alpha_y}} \big], \quad (4)$$

where the parameters a_{jk} and b_{jk} are defined above. In order to define α_x and α_y, we first define $e_x = \text{harmod}(|\Delta_+^x u_{jk}|), |\Delta_-^x u_{jk}|)$, and $e_y = \text{harmod}(|\Delta_+^y u_{jk}|), |\Delta_-^y u_{jk}|)$. Then the parameters α_x and α_y are defined as

$$\begin{aligned}
\alpha_x &= 2 \left[\sqrt{\frac{e_x}{\Delta_+^x u_{jk}}} - 1 \right], \text{if } |\Delta_+^x u_{jk}| \leq |\Delta_-^x u_{jk}| \\
\alpha_x &= 2 \left[1 - \sqrt{\frac{e_x}{\Delta_+^x u_{jk}}} \right] \text{if } |\Delta_+^x u_{jk}| > |\Delta_-^x u_{jk}| \\
\alpha_y &= 2 \left[\sqrt{\frac{e_y}{\Delta_+^y u_{jk}}} - 1 \right], \text{if } |\Delta_+^y u_{jk}| \leq |\Delta_-^y u_{jk}| \\
\alpha_y &= 2 \left[1 - \sqrt{\frac{e_y}{\Delta_+^y u_{jk}}} \right] \text{if } |\Delta_+^y u_{jk}| > |\Delta_-^y u_{jk}| \quad (5)
\end{aligned}$$

We now define the upsampling operator S at the center of half-size pixels in each dimensions as

$$S(x_j(\theta_x), y_k(\theta_y)) = H_{jk}(x_j \theta_x), y_k(\theta_y), \quad (6)$$

where $x_j(\theta_x) = x_j + \theta_x h_x$, and $y_k(\theta_y) = y_k + \theta_y h_y$, and $\theta_x, \theta_y \in [-1/4, 1/4]$. Similarly, the downsampling operator D is defined as $D(v)_{jkl} = \frac{1}{4}[\sum v(x_j(\theta_x), y_k(\theta_y))]$. On account of the higher-order approximation, it is noted that the upsampling criteria is given by $D \circ S(u) = u + O(((h_x h_y))$, and no longer identity. However, from a practical standpoint, the errors are negligible, and the signal is not degraded in a significant way.

3 Results

In this section, we present results of upsampled images by applying the operator specified in Eq. 6. Figure 1 shows upsampled images obtained using bilinear, b-spline, and the edge-preserved upsampling operators. The respective PSNR values are quantified in Table 1. In order to calculate the PSNR, the original images were downsampled using bilinear interpolation and then upsampled using each of the upsampling operators. The edge-preserving piecewise hyperbolic operator not only outperforms bicubic, and b-spline interpolation, but the resulting upsampled images appear sharper, and show better edges. Finally, the edge-preserving operator is also used to upsample color images as shown in Fig. 2. Here, the color image was first separated into luminance (Y), and the two chrominance channels (UV), and each interpolation operator was applied separately to each channel, and then converted back to the RGB colorspace. It is observed that the edge-preserved upsampled images resolve sharpness and detail better compared to the bilinear, and spline based approaches.

Image	**Bilinear**	**Bicubic**	**B-Spline**	**Edge-preserved**
Moon	49.59 dB	51.24 dB	51.37 dB	**51.77 dB**
Math	73.85 dB	77.73 dB	79.16 dB	**82.44 dB**
Text	39.18 dB	40.83 dB	41.38 dB	**43.61 dB**
House	47.02 dB	48.58 dB	48.84 dB	**49.50 dB**

Table 1. Comparison of PNSR ($10 \log(\frac{255^2}{||I_{upsampled} - I_{orig}||^2})$) calculated from the $\mathbb{L}^2$ norm between the upsampled and the original image for each of the interpolation operators.

Fig. 2. From top: Upsampled images ($\times 2$) resulting from bilinear, b-spline, and edge-preserved interpolation applied to the downsampled original image. The same interpolation method is applied separately to each of the luminance and the two chrominance channels.

4 Discussion

The proposed piecewise hyperbolic operator ensures that the reconstructed upsampled functions are smooth inside each pixel, and limits oscillations and jump discontinuities located at the pixel interfaces. The operator is simple to implement, and executes in 0.2 s in MATLAB on an Intel 2.4 GHz platform. We anticipate the usefulness of this operator in routine image processing as well as computer vision tasks, where the preservation of edge quality is of primary importance.

5 References

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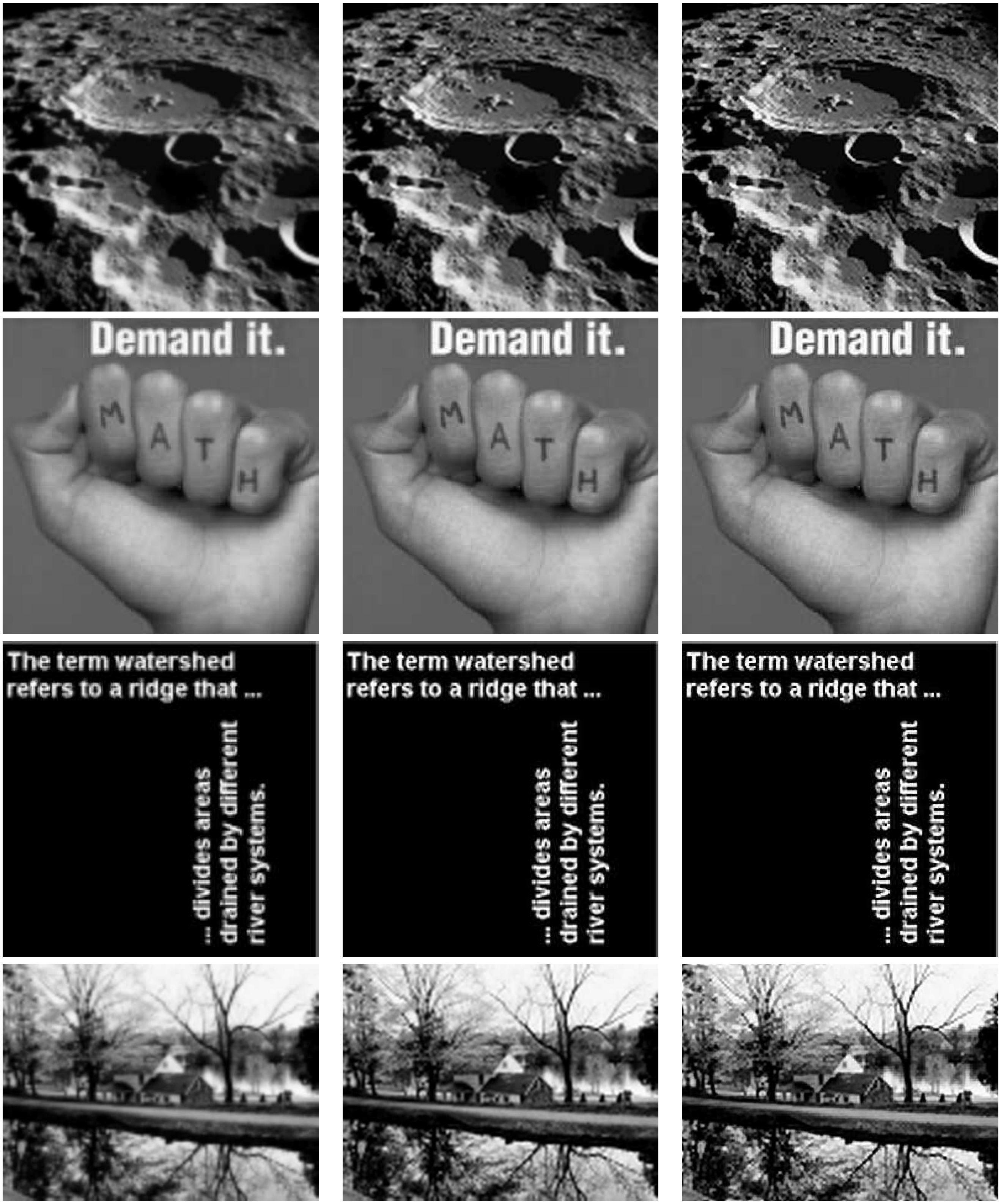

Bilinear **B-spline** **Edge-preserved**

Fig. 1. Upsampled images ($\times 2$) resulting from bilinear, b-spline, and edge-preserving interpolation applied to the downsampled image.