Multispectral Deconvolution of Hurricane Imagery

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Abstract

We develop an approach for reconstruction of high resolution multispectral images from blurry and noisy scenes. This problem arises in the study of hurricanes, among other physically deforming phenomena. Hurricanes are imaged using microwave sensors, which fly aboard research planes and spacecraft and are designed to penetrate through thick clouds to see the structure of a storm. Such images may represent distribution of temperature, water vapor, and cloud liquid water and are valuable for evaluating the storm's internal processes and its strength. Imagery generated using microwave sensors is blurry, noisy, and of low resolution.

1. Introduction

Hurricanes cause catastrophic floods and landslides leading to the loss of property and life. On the other hand, these tropical cyclones are sources for replenishment of the freshwater supply, which is important for the marine life and the ecological environment. However, many aspects of hurricane formation and strength prediction are still unknown. While scientists can usually estimate the location where a powerful storm will hit the land, it is far more difficult to forecast the storm strength. Reconstructed high resolution images will be of value to science, where hurricane formation and strength prediction are major challenges, and to society, which could benefit from more accurate information being used in forecasts of storm strength and development.

2. Notation

We consider $m \times n$ images with l channels represented as a vector, such as

$$u = \begin{bmatrix} u^{(1)} \\ \vdots \\ u^{(l)} \end{bmatrix} \in \mathbb{R}^{lmn},$$

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where $u^{(1)}, \ldots, u^{(l)}$ represent l channels of u.

As in [7, 8, 9], we let $D^{(1)}, D^{(2)} \in \mathbb{R}^{mn \times mn}$ be the first-order forward finite difference operators in the horizontal and vertical directions, respectively. Matrix $D_i \in \mathbb{R}^{2 \times mn}$ contains the ith rows of $D^{(1)}$ and $D^{(2)}$ as its first and second rows, respectively. The total finite difference operator and the discrete gradient of $u^{(c)}$, $1 \le c \le l$, at pixel i are given by

$$D = \begin{bmatrix} D^{(1)} \\ D^{(2)} \end{bmatrix} \in \mathbb{R}^{2mn \times mn},$$

$$D_i u^{(c)} = \begin{bmatrix} \left(D^{(1)} u^{(c)}\right)_i \\ \left(D^{(2)} u^{(c)}\right)_i \end{bmatrix} \in \mathbb{R}^2, \quad i = 1, \ldots, mn, \ c = 1, \ldots, l,$$

respectively. Denote vectors $d_1, d_2 \in \mathbb{R}^{lmn}$, and $d = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \in \mathbb{R}^{2lmn}$. For each pixel $i = 1, \ldots, mn$, denote $\mathbf{d}_i = \begin{bmatrix} (d_1^{(1)})_i \\ (d_2^{(1)})_i \\ \vdots \\ (d_1^{(l)})_i \\ (d_2^{(l)})_i \end{bmatrix} \in \mathbb{R}^{2l}$. Similarly, $b_1, b_2 \in \mathbb{R}^{lmn}$, $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbb{R}^{2lmn}$,

and $\mathbf{b}_i = \begin{bmatrix} (b_1^{(1)})_i \\ (b_2^{(1)})_i \\ \vdots \\ (b_1^{(l)})_i \\ (b_2^{(l)})_i \end{bmatrix} \in \mathbb{R}^{2l}$.

3. Split Bregman Multispectral Deconvolution

Degradation Model. An observed image $f \in \mathbb{R}^{lmn}$ is written as

$$f = Ku + \kappa, \tag{1}$$

where $u \in \mathbb{R}^{lmn}$ is an unknown clean image and $\kappa \in \mathbb{R}^{lmn}$ is the noise function. K is the convolution kernel of the form

$$K = \begin{bmatrix} K_{11} & K_{12} & \cdots & K_{1l} \\ K_{21} & K_{22} & \cdots & K_{2l} \\ \vdots & \vdots & \ddots & \vdots \\ K_{l1} & K_{l2} & \cdots & K_{ll} \end{bmatrix} \in \mathbb{R}^{lmn \times lmn},$$

where $K_{ij} \in \mathbb{R}^{mn \times mn}$, each matrix K_{ii} is the blurring operator within the ith channel, and each matrix K_{ij}, $i \ne j$, is the blurring operator across channels i and j.

We use total variation based regularization in order to recover image u from (1). The bounded variation (BV) norm, measuring the total variation (TV) was originally proposed for image denoising in [6], and had since been used to solve a variety of problems in image

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processing and computer vision. The effectiveness of the BV norm stems from its ability to preserve edges in an image.

Total Variation in Grayscale Case. The total variation (TV) of a grayscale image u can be defined as

$$TV(u) = \int_\Omega ||\nabla u|| dx, \tag{2}$$

where $|| \cdot ||$ is a norm in $\mathbb{R}^2$. A discrete form of (2) is given by

$$TV(u) = \sum_i ||D_i u||. \tag{3}$$

If $|| \cdot ||$ is the 2-norm, (2) defines isotropic TV with a discrete form:

$$TV(u) = \sum_i \sqrt{\left(D^{(1)}u\right)_i^2 + \left(D^{(2)}u\right)_i^2}.$$

If $|| \cdot ||$ is the 1-norm, (2) defines anisotropic TV with a discrete form:

$$TV(u) = \sum_i \left|\left(D^{(1)}u\right)_i\right| + \left|\left(D^{(2)}u\right)_i\right|.$$

We will assume $|| \cdot ||$ denotes the 2-norm.

Multispectral Deconvolution. Letting matrix $M_i \in \mathbb{R}^{2l \times lmn}$ to contain l^2 blocks of size $2 \times mn$, with each diagonal block containing a matrix D_i, and non-diagonal blocks filled with zeros:

$$M_i = \begin{bmatrix} D_i & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & D_i & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & D_i \end{bmatrix} \in \mathbb{R}^{2l \times lmn},$$

the total variation for multichannel images in the discrete form is given as

$$MTV(u) = \sum_i ||M_i u|| = \sum_i \sqrt{||D_i u^{(1)}||_2^2 + \cdots + ||D_i u^{(l)}||_2^2}. \tag{4}$$

The minimization problem for multichannel deconvolution with L_1-norm fidelity term is given as

$$\min_u \sum_i ||M_i u|| + \frac{\mu}{2}||Ku - f||_1, \tag{5}$$

where $\mu > 0$ is a parameter. Here, $M_i u \in \mathbb{R}^{2l}$. Matrix M_i contains the ith rows of $M^{(j)}$, $1 \leq j \leq 2l$, as its rows, where

$$M = \begin{bmatrix} M^{(1)} \\ \vdots \\ \vdots \\ M^{(2l)} \end{bmatrix} = \begin{bmatrix} D & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & D & \ddots & \vdots \\ \vdots & \ddots & \ddots & \mathbf{0} \\ \mathbf{0} & \cdots & \mathbf{0} & D \end{bmatrix} \in \mathbb{R}^{2lmn \times lmn}, \quad \text{with } D = \begin{bmatrix} D^{(1)} \\ D^{(2)} \end{bmatrix} \in \mathbb{R}^{2mn \times mn}.$$

In [7], the authors proposed the alternating minimization algorithm for solving TV-L^2 single-channel deconvolution problems. The algorithm was extended to solve TV-L^2 multichannel deconvolution problems in [8]. In [9], the authors proposed the alternating minimization variable-splitting algorithm for minimizing (5). Also, the Split Bregman algorithm for denoising grayscale images with L_2-norm fidelity term was proposed in [3]. These formulations are related to problems that arise frequently in compressed sensing [1, 2].

Inspired by these methodologies, we will minimize the multispectral deconvolution problem (5), within the Split Bregman minimization framework. We consider the following minimization problem, which is based on a half-quadratic approximation of (5), and where an additional variable $z \in \mathbb{R}^{lmn}$ is introduced to approximate $Ku - f$:

$$\min_{u,\mathbf{d},z} \sum_i ||\mathbf{d}_i||_2 + \frac{\lambda}{2} \sum_i ||\mathbf{d}_i - M_i u - \mathbf{b}_i||_2^2 + \mu ||z||_1 + \frac{\alpha}{2} ||z - (Ku - f) - w||_2^2. \quad (6)$$

Here, λ and α are nonnegative parameters, and variables $\mathbf{b}_i$ and w are chosen through Bregman iterations [10, 5]:

$$\begin{aligned} \mathbf{b}_i &\leftarrow \mathbf{b}_i + (M_i u - \mathbf{d}_i), \\ w &\leftarrow w + (Ku - f - z). \end{aligned}$$

For a fixed u, the minimization problem for $\mathbf{d}_i$ is

$$\mathbf{d}_i^* = \arg\min_{\mathbf{d}_i} \sum_i ||\mathbf{d}_i||_2 + \frac{\lambda}{2} \sum_i ||\mathbf{d}_i - M_i u - \mathbf{b}_i||_2^2.$$

We can explicitly solve the minimization problem for $\mathbf{d}_i$ using a generalized shrinkage formula [7]:

$$\mathbf{d}_i = \max\left(||M_i u + \mathbf{b}_i||_2 - \frac{1}{\lambda}, 0\right) \frac{M_i u + \mathbf{b}_i}{||M_i u + \mathbf{b}_i||_2}.$$

The minimization problem for z is

$$z^* = \arg\min_z \mu ||z||_1 + \frac{\alpha}{2} ||z - (Ku - f) - w||_2^2,$$

with a minimizer given by the one-dimensional shrinkage:

$$z = \max\left\{|Ku - f + w| - \frac{\mu}{\alpha}, 0\right\} \text{sign}(Ku - f + w).$$

For a fixed $\mathbf{d}$ and z, the minimization problem (6) for u is quadratic in u:

$$u^* = \arg\min_u \sum_i ||\mathbf{d}_i - M_i u - \mathbf{b}_i||_2^2 + \frac{\alpha}{\lambda} ||z - (Ku - f) - w||_2^2,$$

and the minimizer is given by the normal equations:

$$\left(M^T M + \frac{\alpha}{\lambda} K^T K\right) u = M^T(b - d) + \frac{\alpha}{\lambda} K^T(f + z - w),$$

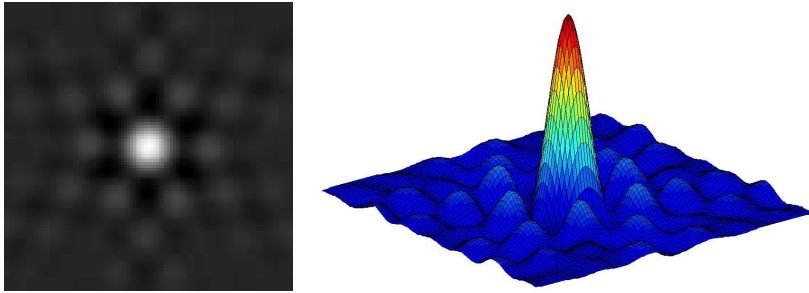Figure 1. One of the microwave channels of a simulated multispectral hurricane image.

Figure 2. The sinc kernel $K(x, y) = \text{sinc}(x)\text{sinc}(y)$.

which are solved using the fast Fourier transform.

An efficient formulation for solving multichannel TV-L^2 deconvolution

$$\min_u \sum_i ||M_i u|| + \frac{\mu}{2}||Ku - f||_2^2$$

within the Split Bregman framework, can similarly be obtained.

4. Results

We tested the method described above on 402 by 402 grayscale hurricane image in Figure 1. This data was simulated using cloud resolving numerical weather prediction model [4]. We used 101 by 101 sinc kernel, defined as $K(x, y) = \text{sinc}(x)\text{sinc}(y)$ and shown on Figure 2 to blur the image. The sinc kernel K takes both positive and negative values and closely characterizes point spread functions of microwave aperture synthesis systems. Compared with out-of-focus and Gaussian blurs, the sinc kernel is significantly more difficult to deconvolve. If images convolved with out-of-focus, Gaussian, and sinc kernels have identical signal-to-noise ratios (SNR), we expect deconvolution result of a sinc-convolved image to have a smaller SNR compared to deconvolutions from other kernels.

Figures 3(a) and (c) show effects of degrading image in Figure 1 with the sinc blur as well as impulse and Gaussian noise, respectively. Figures 3(b) and (d) display deconvolution results obtained with the efficient TV-L^1 and TV-L^2 Split Bregman models, respectively.
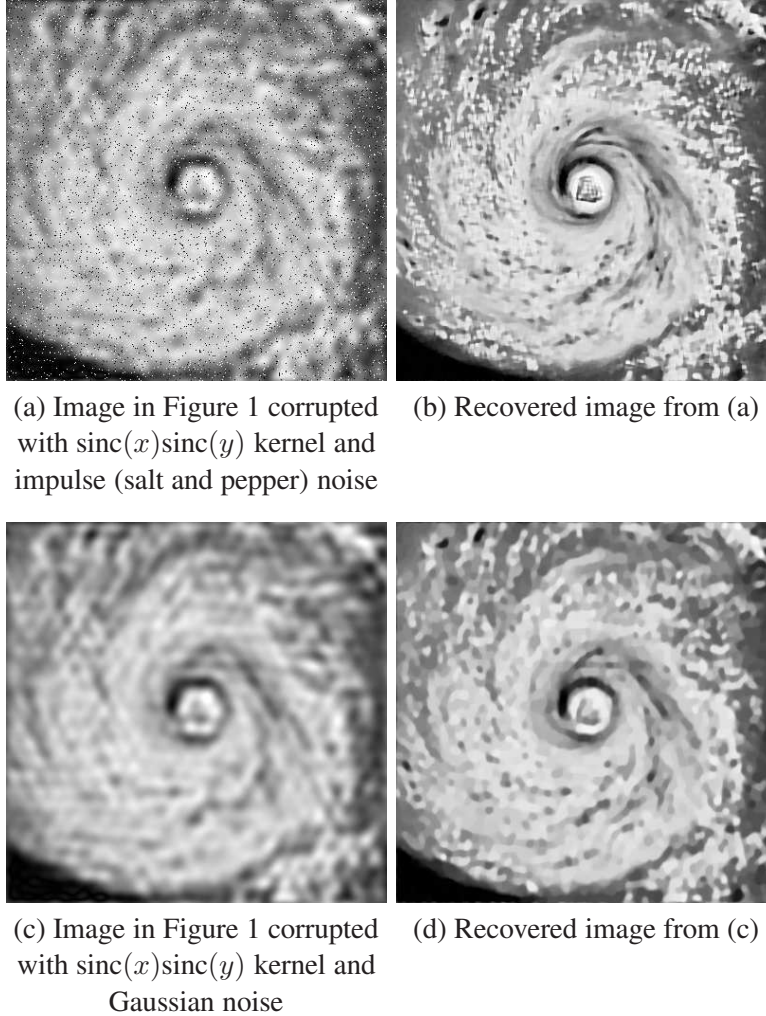
(a) Image in Figure 1 corrupted with sinc(x)sinc(y) kernel and impulse (salt and pepper) noise

(b) Recovered image from (a)

(c) Image in Figure 1 corrupted with sinc(x)sinc(y) kernel and Gaussian noise

(d) Recovered image from (c)

Figure 3. TV-L^1 and TV-L^2 Split Bregman deconvolution of a simulated hurricane image. Images (a) and (b) show a deconvolution result for Figure 1 image corrupted with sinc blur and impulse noise. Images (c) and (d) show a deconvolution result for Figure 1 image corrupted with sinc blur and Gaussian noise.

Acknowledgements

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