Automatic prior shape selection for image segmentation

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Abstract Segmenting images with occluded and missing intensity information is still a difficult task. Intensity based segmentation approaches often lead to wrong results. High vision prior information such as prior shape has been proven to be effective in solving this problem. Most existing shape prior approaches assume known prior shape and segmentation results rely on the selection of prior shape. In this paper, we study how to do simultaneous automatic prior shape selection and segmentation in a variational scheme.

1 Introduction

Image segmentation has many important applications in object recognition, machine learning, medical imaging, etc. In medical imaging for instance, segmentation of anatomical structures is used to help in diagnosis, surgical planning and evaluation. Intensity based image segmentation methods can be classified into region based, edge-based and a combination of these two. Using image intensity information alone however may not lead to desired results when the image to be segmented has significant signal loss, poor image contrast and missing boundaries. Prior shape based approaches are more effective in these cases. Most existing shape based approaches assume the shape prior is given and a misleading prior shape might lead to wrong

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segmentation. We use sparse optimization to automatically select prior shapes from a shape library and simultaneously segment images. The proposed variational approach is able to automatically and adaptively select prior shape which in turn guides segmentation. It is especially beneficial when there are objects with multiple shapes to segment.

The rest of the paper is organized as follows: Section 2 introduces the proposed model. Numerical results are presented in Section 3. Conclusion is drawn in Section 4.

2 Model Description

In this section, we start by reviewing the Ambrosio-Tortorelli approximation of the Mumford-Shah model, then we describe how to apply it to form the shape library. Lastly, we present the proposed segmentation model.

2.1 Ambrosio-Tortorelli Approximation of Mumford-Shah Segmentation Functional

Given an image $g(x)$ defined on an open and bounded set $\Omega \subset \mathbb{R}^2$ satisfying $g \in L^\infty(\Omega)$, Mumford and Shah [1] proposed the following functional for image segmentation

$$F^{MS}(u,S) = \int_{\Omega/S} \left(\alpha|\nabla u|^2 + \beta|u-g|^2\right) dx + \mathcal{H}^1(\Omega).$$

where $\mathcal{H}^1$ is the Hausdorff 1-dimensional measure in $\mathbb{R}^2$, i.e.,

$$\mathcal{H}^1(S) = \sup_{\delta>0} H^1_\delta(S) = \lim_{\delta\to 0} H^1_\delta(S) = \liminf_{\delta\to 0} \left\{ \sum_{i=1}^{\infty} (\text{diam}(U_i))^d : \bigcup_{i=1}^{\infty} U_i \supseteq S, \text{diam}(U_i) < \delta \right\}.$$

The functional is optimized in a weak sense and can be approximated by [2]

$$G^{AT}_\rho(u,v) = \int_\Omega \left[\rho|\nabla v|^2 + \alpha v^2|\nabla u|^2 + \frac{(v-1)^2}{4\rho} + \beta|u-g|^2\right] dx.$$

Then the image segmentation is to find a piecewise C^1 function $u(x)$ and a function $v(x)$, such that $v(x) \to 1$ as $\rho \to 0$ in the $L^2(\Omega)$-topology, i.e.,

$$\lim_{\delta\to 0} \int_\Omega |v-1|^2 dx = 0.$$

Neither Mumford-Shah model nor its Ambrosio-Tortorelli approximation can work well for images with missing or occluded edge information. Shape prior is required in this case to obtain a complete segmentation. We use sparse optimization to search for prior shapes adapt to images automatically.

2.2 Shape Descriptor Library

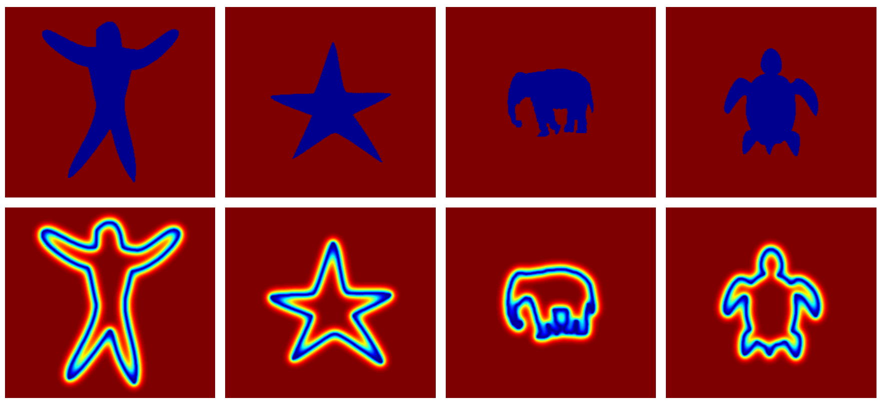

Fig. 1 Examples of silhouette images (top row) and their edge strength functions (bottom row).

We start by reviewing edge strength functions presented in [3] to form a library. Then we will describe how to use these functions to form our shape libraries. These edge strength functions have distance function look and provide richer information than the binary silhouette images (see Fig. 1). For notational simplicity, we use the same notation to interchangeably represent a matrix and its vectorized version. For the rest of the paper, we consider discrete models. For instance,

$$\int_{\Omega} \left(\rho |\nabla v|^2 + \frac{(v-1)^2}{4\rho} \right) dx \tag{1}$$

is discretized as $\rho \|\nabla v\|_2^2 + \frac{\|v-1\|_2^2}{4\rho}$.

For each binary image, we compute its edge strength function based on the following diffusion model with Dirichlet boundary condition

$$v_i = \operatorname*{argmin}_v \rho \|\nabla v\|_2^2 + \frac{\|v-1\|_2^2}{4\rho}, \quad v = 0 \text{ on the boundary of the } i\text{th binary image.} \tag{2}$$

Given a library

$$A = \begin{bmatrix} | & | & & | \\ v_1 & v_2 & \cdots & v_N \\ | & | & & | \end{bmatrix},$$

our goal is to learn a prior shape v such that

$$v = As + w + e = \sum_{i=1}^{N} s_i v_i + w + e,$$

where w is discrepancy and e is random Gaussian noise. Considering the sparsity of s and edge-like characteristic of w, we propose the following model:

$$\min_{s,w} \|\nabla w\|_1 + \beta \|s\|_0 \quad \text{subject to} \quad \|As + w - v\|_2 \leq \varepsilon,$$

where $\alpha, \beta > 0$ are parameters, and ε is the standard deviation of the error. By converting into the unconstrained minimization problem, the above model reads as

$$\min_{s,d} \frac{1}{2} \|As + w - v\|_2^2 + \alpha \|\nabla w\|_1 + \beta \|s\|_0 .$$

Since the ℓ_0 problem is NP-hard, we make a relaxation and solve the following ℓ_1 problem

$$\min_{s,d} \frac{1}{2} \|As + w - v\|_2^2 + \alpha \|\nabla w\|_1 + \beta \|s\|_1 .$$

The reason that we use the edge strength function for shape rather than any other informative indicator function (e.g., signed distance function) is that it has a natural connection to the segmentation problem via Mumford and Shah. Note that the edge strength function is nothing but the minimizer of (1). In the previous section we have explained that the edge strength function approaches to the edge indicator in the $L^2(\Omega)$-topology as $\rho \to 0$. Interestingly, as we increase ρ, edge localization weakens and v begins to act as a *morphology coder*: 1) v value at a domain point is a monotonically decaying function of the distance from the point to the domain boundary (the edge set); 2) the level curves of v are curvature dependent erosions of the domain boundary [3]. Thus, in our model, unlike many other shape prior based segmentation models, we do not distinguish inside and outside in the intermediate steps.

2.3 Proposed Segmentation Model

Given a reference image $g(x)$, we propose the following segmentation model:

$$\min_{u,v,s,d,h} \frac{1}{2} \|u - g\|_2^2 + \frac{\alpha}{2} \|v \cdot \nabla u\|_2^2 + \frac{\rho}{2} \|\nabla v\|_2^2 + \frac{\|v-1\|_2^2}{8\rho} + \beta \|\nabla w\|_1 + \tau \|s\|_1 + W(h)$$
$$\text{subject to} \quad As + w = v(h).$$

where $\cdot$ represents point-wise product, $W(h)$ is a regularization term with respect to h. Note that to make variables consistent in the above model $\|\nabla u\|_2^2$ and $\|\nabla v\|_2^2$ are the discretized versions of $\int_\Omega |\nabla u|^2 dx$ and $\int_\Omega |\nabla v|^2 dx$. Typically $W(h)$ is set as $\|\nabla h\|_2^2$.

The associated Lagrangian function is

$$\begin{aligned} L(u,h,v,s,w,t) = \frac{1}{2}\|u-g\|_2^2 + \frac{\alpha}{2}\|v\cdot\nabla u\|_2^2 + \frac{\rho}{2}\|\nabla v\|_2^2 + \frac{\|v-1\|_2^2}{8\rho} \\ + \beta\|\nabla w\|_1 + \tau\|s\|_1 + W(h) + \frac{\gamma}{2}\|As+w-v(h)-t\|_2^2 \end{aligned}$$

where t is the scaled Lagrange multiplier and γ is a positive parameter. Since v and h are related and inseparable, we cannot directly apply the ADMM to solve the above model. As such, we consider the following modified ADMM with approximate subproblems:

$$\begin{cases} u^{k+1} = \operatorname*{argmin}_u \frac{1}{2}\|u-g\|_2^2 + \frac{\alpha}{2}\left\|v^k\cdot\nabla u\right\|_2^2, \\ v^{k+1} = \operatorname*{argmin}_v \frac{\alpha}{2}\left\|v\cdot\nabla u^{k+1}\right\|_2^2 + \frac{\rho}{2}\|\nabla v\|_2^2 + \frac{\|v-1\|_2^2}{8\rho} + \frac{\gamma}{2}\left\|As^k+w^k-v(h^k)-t^k\right\|_2^2 \\ h^{k+1} = \operatorname*{argmin}_h \frac{\gamma}{2}\left\|As^k+w^k-v^{k+1}(h)-t^k\right\|_2^2 + W(h) \\ s^{k+1} = \operatorname*{argmin}_s \tau\|s\|_1 + \frac{\gamma}{2}\left\|As+w^k-v^{k+1}(h^{k+1})-t^k\right\|_2^2 \\ w^{k+1} = \operatorname*{argmin}_d \beta\|\nabla w\|_1 + \frac{\gamma}{2}\left\|As^{k+1}+w-v^{k+1}(h^{k+1})-t^k\right\|_2^2 \\ t^{k+1} = t^k + \gamma(v^{k+1}(h^{k+1}) - (As^{k+1}+w^{k+1})) \end{cases} \tag{3}$$

The u-subproblem can be solved by applying the negative gradient flow

$$\frac{du}{dt} = -2(u-g) + \alpha\,\mathrm{div}\big((v^k)^2\nabla u\big). \tag{4}$$

Likewise, the v-subproblem can be solved iteratively. The h-subproblem turns out to be a registration problem. Moreover, the s-subproblem and the w-subproblem are Lasso problems which can be directly solved by applying ADMM [4, 5].

3 Experiments

In this section, we show two numerical experiments to validate our proposed method. By the assumption that the desired edge strength function v is a linear combination of atoms in the library A, the atoms have to be linearly independent which ensures the unique representation of v in the column space of A. In addition, to avoid the interruptions of background during the learning process, we also restrict the data fidelity term in the s-subproblem to the union of shape interiors associated with atoms, which can be done by introducing the corresponding mask. In all our experiments, the library consists of 5 independent atoms which are generated by applying the model (2) to 5 binary shapes (see Fig. 2). The parameters for both experiments are set as $\rho = 8$, $\alpha = \gamma = 1$, and $\beta = 10^{-2}$.

Fig. 2 Atoms in the library used in the experiments

At the first experiment, we test an image where a star is partially occluded by the background rectangles. After running 13 iterations, the desired atom corresponding to the star shape is learned from the library. The input image, the obtained edge strength function v and the extracted boundary by thresholding v with 0.015 respectively are shown in Fig. 3. This example shows that the proposed algorithm is able to find a matching shape from the library. At the second experiment, we test an image where a star with missing parts is contaminated by uniformly distributed Gaussian noise with zero mean and standard deviation $\sigma = 0.8$. After 60 iterations, the desired edge strength function v is obtained with noise reduction. The extracted boundary by thresholding v with 0.04 is shown in Fig. 4. One can see that the proposed method has a potential to supplement the insufficiency of input data by learning a prior shape from the library. The resultant edge strength images can be further processed to obtain sharp boundaries of the objects by thresholding or other more sophisticated algorithms.

Fig. 3 From left to right: the input image which has a star occluded partially by the background, the output edge strength function v, and the extracted boundary.

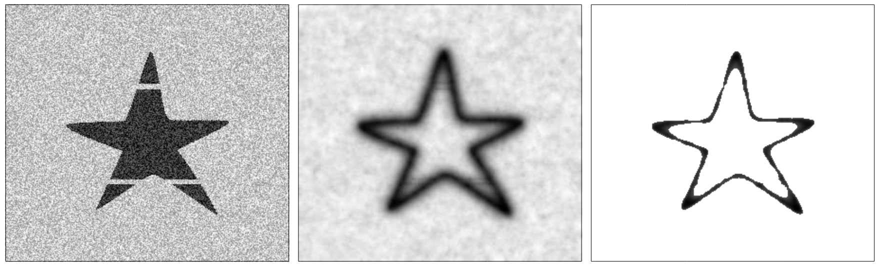

Fig. 4 From left to right: the input image which has a star in a noisy background with missing parts, the output edge strength function v, and the extracted boundary.

4 Conclusion

Shape prior plays an important role in segmenting images with occlusive and missing information. In this paper, we used edge strength functions as atoms of a library and applied sparse optimization methods to automatically and adaptively search for a shape prior from the library to guide segmentation in a variational scheme. Numerical experiments show that the proposed approach has some potentials in segmenting images with missing information, random noise and structure noise.

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