Energy-efficient Velocity Control for Massive Rotary-Wing UAVs: A Mean Field Game Approach

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***Abstract*—When a disaster happens in the metropolitan area, the wireless communication systems are highly affected, degrading the efficiency of the search and rescue (SAR) mission. An emergent wireless network must be deployed quickly and efficiently to preserve human lives. Teams of low-altitude rotary-wing unmanned aerial vehicle (UAVs) is preferable to be utilized as an on-demand temporal wireless network because they are generally faster to deploy, flexible to reconfigure, and able to provide better communication services with short line-of-sight links. However, rotary-wing UAVs' limited on-board batteries require that they need to recharge and reconfigure frequently during a mission. Therefore, we formulate the velocity control problem for massive rotary-wing UAVs as a Schrödinger bridge problem which can describe the frequent reconfiguration of UAVs. Then we transform it into a mean field game and solve it with the G-prox primal dual hybrid gradient (PDHG) method. Finally, we show the efficiency of our algorithm and analyze the influence of wind dynamics with numerical results.**

I. Introduction

Large-scale natural catastrophe in the metropolitan areas can inflict unimaginable losses of human lives and property [1]. Efficiently and quickly conducted search and rescue (SAR) operations are needed to preserve human lives. However, as the communication systems are always highly destroyed after disaster, efficient rescue mission is extremely reduced. Different types of communication systems can be provided for SAR, such as terrestrial communications, high-altitude platforms, and low-altitude unmanned aerial vehicles (UAVs), among which the on-demand wireless communication systems consisting of teams of UAVs are preferable because UAVs are generally faster to deploy, flexible to reconfigure, and able to provide better communication services with short line-of-sight links [2].

Different types of UAVs can be utilized to establish the temporal communication network for SAR, such as blimps, balloons, fixed-wing and rotary-wing UAVs. Compared with other UAVs, rotary-wing UAVs are appealing because of its low price and ability to hover [1]. However, rotary-wing UAVs also face a common challenge as the other UAVs because they can remain airborne only for 15-20 minutes given their limited on-board energy, which means multiple rounds of recharging and then reforming specified wireless networks are needed. Thus, their movements in each round must be optimized to complete more successful missions. However, the movement control for massive rotary-wing UAVs are difficult because of the frequent interactions between them and the lack of centralized controller.

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Hao Gao and Wonjun Lee share equal contributions on this paper

Regarding above disaster scenarios and energy limitation challenges faced by UAV-supported wireless networks, we formulate the movement control for massive rotary-wing UAVs as a Schrödinger bridge problem [3], [4] for the following reasons: (i) it can describe the frequent switch between different distributions of the large number of UAVs through a single stochastic process. (ii) it allows the differential constraint, which can describe the relationship between UAVs' velocities and locations. Then we propose a mean field game theoretic approach to solve the Schrödinger bridge problem because the generic UAV can determine its velocity by reacting to the collective behavior of all other UAVs, i.e., mean field, instead of to every other UAV, which reduce the complexity of the problem significantly. Specifically, UAVs can determine their velocities through solving a partial differential equation (PDE), the Hamilton-Jacobi-Bellman (HJB) equation, and the mean field information. After the UAVs make their decisions, the teams of UAVs can eovlve to another distribution by solving another PDE, the Fokker-Planck-Kolmogorov (FPK) equation [5]. The evolution of velocities and the mean field can reach an equilibrium, which generate the optimal velocity control for all the UAVs.

Related Work [6] focuses on optimizing the spectral efficiency without any major increase in the energy consumption while [7] proposes a 3-D placement algorithm for UAV base stations which can optimize the coverage area with minimum transmit power. Both of them solve the UAV-aided wireless communication problem under the sparse deployment scenario. [8] considers the trajectory optimization problem for rather dense deployment of UAVs and propose an energy-efficient flocking algorithm which can minimize the power consumption per downlink rate in one dimension case. However, [8] utilizes a simplified power consumption model consisting of kinetic energy and constant transmission power. In contrast, [9] derives a practical power consumption model for rotary-wing UAV but only consider the trajectory optimization problem for a single rotary-wing UAV.

In summary, the main contributions in this paper are:

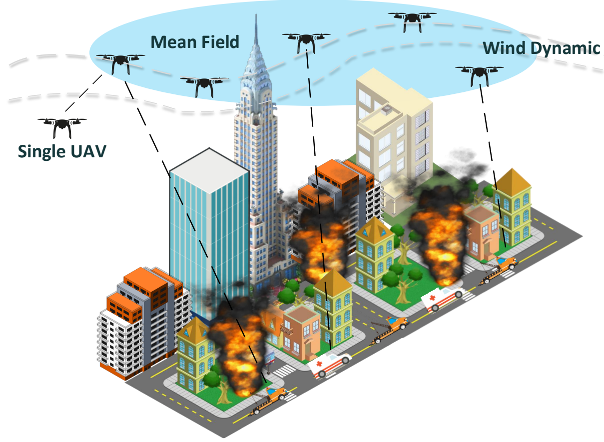


Fig. 1: Emergent communication with teams of UAVs after disaster

- We formulate the velocity control problem for massive rotary-wing UAVs in a two-dimension plane as a Schrödinger bridge problem that can describe the relationship between velocity and location, as well as the frequent reconfiguration of massive UAVs during the disaster.
- We propose a mean field game approach to solve the Schrödinger bridge problem. Specifically, we reformulate the Schrödinger bridge problem as a mean field game and solve it with G-prox primal dual hybrid gradient (PDGH) [10].
- We improve the energy efficiency around 70% for the worst case, i.e., the highest wind variance and analyze the influence of wind dynamics on the energy consumption with numerical results.

The rest of this paper is organized in the following way. In Section II, we model the energy consumption of massive rotary-wing UAVs and formulate the velocity control problem as a Schrödinger bridge problem. In Section III, we propose a mean field game approach to obtain the optimal velocity control, which can minimize the the energy consumption for each UAV. Section IV shows the performance of our algorithm with numerical results, and Section V draws the conclusion.

II. Modeling and Formulating

We consider the scenario when a team of U rotary-wing UAVs are going to provide emergent wireless communication service for several SAR teams in the metropolitan area after a disaster as shown in Fig. 1. U UAVs are flying at the same altitude and thus the coordinate of the ith UAV is $\mathcal{X}_i(t) \in \mathcal{R}^2$. The speed of each UAV $\mathcal{V}_i(t) \in \mathcal{R}^2$ is affected by its location as well as the wind and we assume that the wind dynamics follow the Ornstein-Uhlenbeck process based on [11]. Then we represent the dynamic of the ith UAV's location as

$$d\mathcal{X}_i(t) = (\mathcal{V}_i(t) + A)dt + \eta_A dB_i(t), \tag{1}$$

where A is the average wind velocity, $\eta_A > 0$ is the wind velocity variance and $B_i(t)$ is the standard Brownian motion, which is identical and independent among all UAVs. The initial distribution of the location $\mathcal{X}_i(t)$ is generated by the known distribution $\rho^0 \in \mathbb{R}^2$, i.e., $\mathcal{X}_i^0 \sim \rho^0$. The team of UAVs are going to reform another location distribution $\rho^1 \in \mathbb{R}^2$, which is the location distribution of the SAR teams in a unit time duration. Thus we have $\mathcal{X}_i^1 \sim \rho^1$.

The energy consumption of rotary wing UAVs consists of the transmission energy and the propulsion energy. As the propulsion energy is highly related to the UAV's velocity, we regard it as the main energy consumption and assume the transmission energy of each UAV is a constant when we consider the velocity control problem. The propulsion energy of a rotary-wing UAV is mainly composed of the blade profile energy and the parasite energy [9]. The blade profile energy is utilized to overcome the blade profile drag while the parasite energy is consumed by fuselage drag. The propulsion energy for the ith rotary-wing UAV is computed by

$$\mathcal{P}_i(\mathcal{V}_i) = \underbrace{P_0\left(1 + \frac{3\|\mathcal{V}_i\|^2}{U_{tip}^2}\right)}_{\text{blade profile}} + \underbrace{\frac{1}{2}d_0\rho_a sB\|\mathcal{V}_i\|^3}_{\text{parasite}}, \tag{2}$$

where P_0 is the constant denoting the blade profile energy in hovering status, U_{tip} represents the speed of the rotor blade, d_0 and s are the fuselage drag ratio and rotor solidity, respectively, ρ_a and B represents the air density and rotor disc area, respectively.

In summary, the velocity control problem for the ith UAV can be formulated as the following Schrödinger bridge problem:

$$\inf_{\mathcal{V}_i} \int_0^1 E_{\mathcal{X}_i(t)\sim\rho^t}\mathcal{P}_i(\mathcal{V}_i) \quad dt \tag{3}$$

$$\text{s.t.} \quad d\mathcal{X}_i(t) = (\mathcal{V}_i(t) + A)dt + \eta_A dB_i(t),$$

$$\mathcal{X}_i^0 \sim \rho^0, \mathcal{X}_i^1 \sim \rho^1,$$

where $\mathcal{P}_i(\mathcal{V}_i)$ is the propulsion power consumption defined in (2), the stochastic differential equation is the temporal dynamic of location $\mathcal{X}_i(t)$ defined in (1), ρ^0 and ρ^1 are the initial and final location distribution of $\mathcal{X}_i(t)$, respectively.

III. Mean Field Game Approach

In this section, we find the optimal velocity control for U rotary-wing UAVs which can minimize the energy consumption of each UAV when it travels from the initial location to the target location. Specifically, we first reformulate the Schrödinger bridge problem in (3) for massive UAVs into a mean field game problem in Subsection III-A, and then we propose the energy-efficient velocity control algorithm to solve the mean field game in Subsection III-B.

A. Mean Field Game Formulation

In this subsection, we reformulate the Schrödinger bridge problem in (3) for massive rotary-wing UAVs as a mean field game. As we consider the reaction of a given UAV to the collective behavior of all other UAVs, we now drop the index i of location $\mathcal{X}_i$, velocity $\mathcal{V}_i$ and its corresponding power consumption $\mathcal{P}_i$. Denote

$$\int_W \rho(t,x)dx = Pr(\mathcal{X} \in W), \tag{4}$$

where W is a measurable area on the two dimension plane on which the UAVs are flying and Pr is the probability that the given UAV appears in the area W. Based on (4), the objective function in (3) can be represented in terms of the UAV densities ρ^t as follows:

$$\int_0^1 E_{\mathcal{X}^t \sim \rho^t} \mathcal{P}(\mathcal{V}) dt = \int_0^1 \int_{\mathcal{R}^2} \mathcal{P}(\mathcal{V}) \rho(t,x) dxdt. \tag{5}$$

Then ρ^t satisfies the forward transition equation of x^t, i.e., the FPK equation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho(\mathcal{V} + A)) - \eta_A \triangle \rho = 0, \tag{6}$$

which describes the time evolution of the general location distribution ρ of all UAVs (the mean field) when the given UAV has changed its velocity. To this end, we are ready to give the following mean field game formulation for (3).

Proposition 1 (Mean field game Formulation)**.** *The mean field game formulation for* (3) *is [12]:*

$$\inf_{v,\rho} \int_0^1 \int_{\mathbb{R}^2} \mathcal{P}(v)\rho(t,x) \quad dxdt, \tag{7}$$

$$\text{s.t.} \quad \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho(v + A)) - \eta_A \triangle \rho = 0,$$

$$\rho(0,\cdot) = \rho^0, \rho(1,\cdot) = \rho^1,$$

where the infimum is taken over continuous unnormalized density functions $\rho : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$ *[13] and the given UAV's velocity* $\mathcal{V} : [0,1] \times \mathbb{R}^2 \to \mathbb{R}^2$*, the PDE in the first constraint is the FPK equation given in* (6)*,* ρ^0 *and* ρ^1 *are the initial and target location distribution of UAVs, respectively.*

Remark 1. Our formulation is a generalized Schrödinger bridge problem, in which the objective function in (3) is computed with fixed initial and terminal densities. If we further relax the constraint on the terminal density, our formulation is exactly the mean field game. We will leave this general formulation in a future work.

B. Energy-efficient Velocity Control Algorithm

In this subsection, we solve the mean field game problem in (7) and propose the energy-efficient velocity control algorithm which can minimize the energy consumption when the massive rotary-wing UAVs fly from the initial locations to the target locations.

We solve (7) by solving its Lagrangian dual problem and thus we first give the definition of the Legendre transform and dual problem formulation as follows.

Definition 1 (Legendre transform)**.** *Consider a function* $f : \mathbb{R}^2 \to \mathbb{R}$ *and* f *is a convex function. The Legendre transform of a function* f*, denoted by* $f^* : \mathbb{R}^2 \to \mathbb{R}$*, is defined as*

$$f^*(y) = \sup_{x \in \mathbb{R}^2} x \cdot y - f(x).$$

Proposition 2 (Dual formulation)**.** *The dual formulation for the mean field game problem in* (7) *is:*

$$\sup_{\Phi} \int_{\mathbb{R}^2} \Phi(1,x)\rho(1,x) - \Phi(0,x)\rho(0,x) \ dx, \tag{8}$$

$$\text{s.t.} \quad \partial_t \Phi + \mathcal{H}(\nabla \Phi) + \eta_A \triangle \Phi \leq 0, x \in \mathbb{R}^2, t \in (0,1).$$

Here, $\Phi : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$ *is a dual variable and the constraint is the HJB equation where* $\mathcal{H}$ *is a Legendre transform of* $\mathcal{P}$*.*

$$\begin{aligned}\mathcal{H}(x,p) =& \frac{1}{24} c_0 \left(-c_1 + \sqrt{c_1^2 + 4c_0\|p\|}\right)^3 \\ &+ \frac{1}{8} c_1 \left(-c_1 + \sqrt{c_1^2 + 4c_0\|p\|}\right)^2 - p_0 + p \cdot A,\end{aligned}$$

where $c_0 = \frac{3}{2} d_0 \rho s B$ *and* $c_1 = \frac{6p_0}{U_{tip}^2}$*.*

Proof. By introducing a dual variable $\Phi : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$, we can reformulate the minimization problem into a saddle problem. Assuming the duality gap is zero we can interchange inf and sup. The proof is given in (9). □

Let $\mathcal{V} = \frac{m}{\rho} - A$ where $m : [0,1] \times \mathbb{R}^2 \to \mathbb{R}^2$ is a Borel vector field. The mean field game problem in (7) is equivalent to the following saddle problem [14], [15]:

$$\inf_{m,\rho} \sup_{\Phi} \mathcal{L}(m, \rho, \Phi) \tag{10}$$

where $\Phi : [0,1] \times \mathbb{R}^2 \to \mathbb{R}$ is a dual variable for the minimization problem and a Lagrangian $\mathcal{L}$ is defined as follows:

$$\begin{aligned}\mathcal{L}(m,\rho,\Phi) =& P_0 \left(\rho + \frac{3\|m - \rho A\|^2}{U_{\text{tip}}^2 \rho}\right) + \frac{d_0 \rho_a s B \|m - \rho A\|^3}{2\rho^2} \\ &+ \Phi(\partial_t \rho + \nabla \cdot m - \eta_A \Delta \rho).\end{aligned} \tag{11}$$

We implement the G-prox PDHG [10] to solve the saddle problem due to its stability and faster speed to converge.

$$\begin{cases} \rho^{k+1} = & \arg\min_\rho \mathcal{L}(m^k, \rho, \Phi^k) + \frac{1}{\tau}\|\rho - \rho^k\|_{L^2}^2, \\ m^{k+1} = & \arg\min_m \mathcal{L}(m, \rho^{k+1}, \Phi^k) + \frac{1}{2\tau}\|m - m^k\|_{L^2}^2, \\ \Phi^{k+1} = & \arg\max_\Phi \mathcal{L}(2m^{k+1} - m^k, 2\rho^{k+1} - \rho^k, \Phi) \\ & -\frac{1}{2\sigma}\|\Phi - \Phi^k\|_{H^1}^2, \end{cases} \tag{12}$$

where τ, σ are two small step sizes, $\|\rho - \rho^k\|_{L^2}^2 = \int_0^1 \int_{\mathbb{R}^2} |\rho - \rho^k|^2 dxdt$, $\|m - m^k\|_{L^2}^2 = \int_0^1 \int_{\mathbb{R}^2} \|m - m^k\|^2 dxdt$ and

$$
\begin{aligned}
&\inf_{\mathcal{V},\rho}\sup_{\Phi}\int_0^1\int_{\mathbb{R}^2}\mathcal{P}(\mathcal{V})\rho+\Phi(\partial_t\rho+\nabla\cdot(\rho(\mathcal{V}+A))-\eta_A\Delta\rho)dxdt\\
=&\sup_{\Phi}\inf_{\mathcal{V},\rho}\int_0^1\int_{\mathbb{R}^2}\mathcal{P}(\mathcal{V})\rho+\Phi(\partial_t\rho+\nabla\cdot(\rho(\mathcal{V}+A))-\eta_A\Delta\rho)dxdt\\
=&\sup_{\Phi}\inf_{\mathcal{V},\rho}\int_0^1\int_{\mathbb{R}^2}\mathcal{P}(\mathcal{V})\rho-\partial_t\Phi\rho-\nabla\Phi\cdot(\rho(\mathcal{V}+A))-\eta_A\Delta\Phi\rho dxdt+\int_{\mathbb{R}^2}\Phi(1,x)\rho(1,x)-\Phi(0,x)\rho(0,x)dx\\
=&\sup_{\Phi}\inf_{\mathcal{V},\rho}\int_0^1\int_{\mathbb{R}^2}-\rho(\mathcal{V}\cdot\nabla\Phi-\mathcal{P}(\mathcal{V}))-\nabla\Phi\cdot\rho A-\partial_t\Phi\rho-\eta_A\Delta\Phi\rho dxdt+\int_{\mathbb{R}^2}\Phi(1,x)\rho(1,x)-\Phi(0,x)\rho(0,x)dx\\
=&\sup_{\Phi}\inf_{\rho}\int_0^1\int_{\mathbb{R}^2}-\sup_{\mathcal{V}}\rho(\mathcal{V}\cdot\nabla\Phi-\mathcal{P}(\mathcal{V}))-\nabla\Phi\cdot\rho A-\partial_t\Phi\rho-\eta_A\Delta\Phi\rho dxdt+\int_{\mathbb{R}^2}\Phi(1,x)\rho(1,x)-\Phi(0,x)\rho(0,x)dx\\
=&\sup_{\Phi}\inf_{\rho}\int_0^1\int_{\mathbb{R}^2}\rho(-\mathcal{P}^*(\nabla\Phi)-\partial_t\Phi-\eta_A\Delta\Phi-\nabla\Phi\cdot A)dxdt+\int_{\mathbb{R}^2}\Phi(1,x)\rho(1,x)-\Phi(0,x)\rho(0,x)dx\\
=&\sup_{\Phi}\left\{\int_{\mathbb{R}^2}\Phi(1,x)\rho(1,x)-\Phi(0,x)\rho(0,x)dx:\partial_t\Phi+\mathcal{H}(\nabla\Phi)+\eta_A\Delta\Phi\le 0\right\}.
\end{aligned}
\tag{9}
$$

$\|\Phi-\Phi^k\|^2_{H^1}=\int_0^1\int_{\mathbb{R}^2}(\partial_t\Phi-\partial_t\Phi^k)^2+\|\nabla\Phi-\nabla\Phi^k\|^2dxdt.$

To solve the first iteration in (12), we need to differentiate the equation with respect to ρ

$$
\begin{aligned}
&P_0-\frac{3P_0\|m-\rho A\|^2}{U_{\text{tip}}^2\rho^2}-\frac{6P_0(m-\rho A)\cdot A}{U_{\text{tip}}\rho}\\
&-d_0\rho_a sB\frac{\|m-\rho A\|^3}{\rho^3}-\partial_t\Phi-\eta_A\Delta\Phi+\frac{1}{\tau}(\rho-\rho^k)\\
&-3d_0\rho_a sB\frac{\|m-\rho A\|(m-\rho A)\cdot A}{2\rho^2}=0.
\end{aligned}
\tag{13}
$$

(13) is a 4th order polynomial which doesn't have an explicit solution. We use Newton's method [16] to find the positive root for (13).

To solve the second iteration in (12), we differentiate it with respect to m.

$$
\begin{aligned}
&\frac{6P_0(m-\rho A)}{U_{\text{tip}}^2\rho}+\frac{3d_0\rho_a sB\|m-\rho A\|(m-\rho A)}{2\rho^2}\\
&-\nabla\Phi+\frac{1}{\tau}(m-m^k)=0.
\end{aligned}
\tag{14}
$$

We use Newton's method to find the solution for (14).

The third iteration in (12) can be solved easily. Again, by differentiating the equation with respect to Φ:

$$
\partial_t\rho+\nabla\cdot m-\eta_A\Delta\rho-\frac{1}{\sigma}(-\Delta)(\Phi-\Phi^k)=0 \tag{15}
$$

Solving (15) for Φ, we get an explicit solution for Φ^{k+1}.

$$
\Phi^{k+1}=\Phi^k+\sigma(-\Delta_{t,x})^{-1}(\partial_t\rho+\nabla\cdot m-\eta_A\Delta\rho)
$$

In conclusion, the energy-efficient velocity control algorithm is given in Algorithm 1.

IV. Simulation Result

In this section, we conduct a comprehensive experiment based on Algorithm 1 and manifest the energy efficiency of our approach through numerical simulation results.

Algorithm 1 Energy-efficient velocity control algorithm

Input: initial and target densities ρ^0, ρ^1; average wind velocity A; step size τ, σ; blade profile energy P_0, tip speed U_{tip}, fuselage drag ratio d_0, rotor solidity s, air density ρ_a, rotor disc area B.

Output: the optimal velocity control $\mathcal{V}^\star=\frac{m^\star}{\rho^\star}-A$

1: Find feasible m^0 and ρ^0
2: **for** $k=1,2,\cdots$ while not converged **do**
3: solve (13), (14), (15) with the Newton's method
4: update ρ^k, m^k, and Φ^k through (12)
5: **end for**

A. Simulation Parameters

We assume that $U=500$ UAVs are flying above a $10km\times 10km$ square disaster area. For the convenience of computation, we regard this as a 1×1 unit area. The initial location of UAVs are generated by the following uniform distribution:

$$
\rho_0(x_1,x_2)=1,\quad (x_1,x_2)\in\Omega,
$$

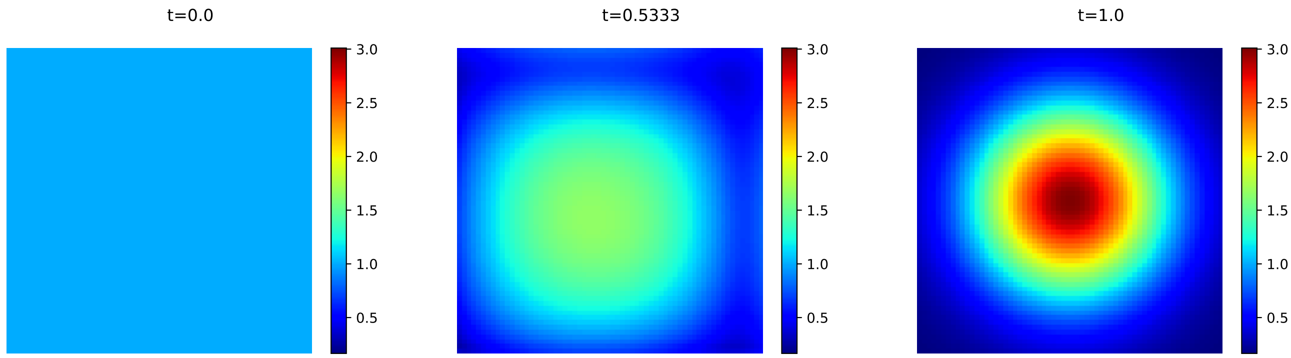
where $\Omega=[0,1]\times[0,1]$ is the unit area. These UAVs are going to provide communication services for SAR teams which have a Gaussian distribution in the same area, i.e., the target the distribution of the UAVs locations are generated by:

$$
\rho_1(x_1,x_2)=\exp\left(-\frac{(x_1-0.5)^2+(x_2-0.5)^2}{0.1}\right),
$$

where $(x_1,x_2)\in\Omega$. Other important parameters after scaling are given in Table I.

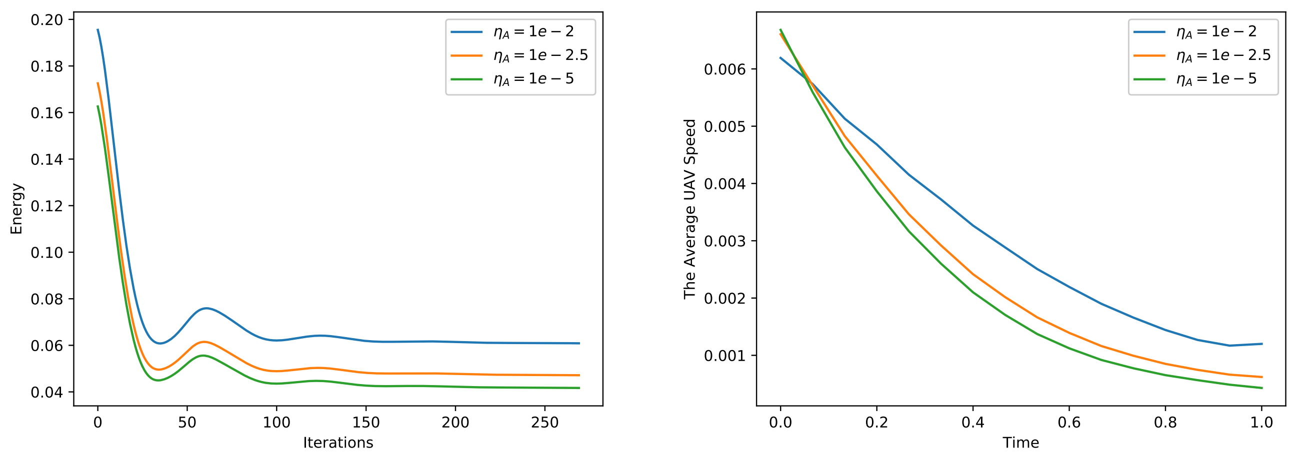
B. Energy Efficiency under Different Wind Dynamics

In Fig. 2, we show the time evolution of UAVs distribution. Brighter color represents higher density of UAVs in that area.



(a) UAVs density at t=0.0 (b) UAVs density at t=0.5333 (c) UAVs density at t=1.0

Fig. 2: Time evolution of UAVs distribution



(a) Energy Convergence (b) Optimal velocity control

Fig. 3: Effect of wind variance

TABLE I: Simulation Parameters

Parameter	Single Resource
blade profile power P_0 (watt)	1×10^{-7}
air density ρ_a (kg/m^3)	1.225
fuselage drag ratio d_0	0.3
rotor solidity s	0.05
rotor disc area B (m^2)	0.01
tip speed U_{tip} (m/s)	3.16×10^{-4}

During the unit time, UAVs move from initial locations (Fig. 2a) to the target locations (Fig. 2c). In Fig. 2b, there is a shift of UAVs distribution to the center of the unit area. The underlying reason for this is that UAVs tend to assemble according to the direction of wind in order to save energy.

In Fig. 3a, we show the convergence behavior of the total energy consumption of UAVs. Without an efficient velocity control, the energy consumption under different wind variances are relatively high. Energy consumption drops very fast after we apply a velocity control, even though it's not optimal. After around 100 iterations when we obtain an optimal velocity control for UAVs, the total energy efficiency is improved by around 70% for the worst case ($\eta_A = 1e-2$) and around 77% for the best case ($\eta_A = 1e-5$). In Fig. 4, the energy consumption increases almost linearly with respect to the increase of wind velocity.

In Fig. 3b, we show UAVs optimal velocity control obtained by the Algorithm 1. The initial average speed of UAVs are high and it gradually decreases when they approach their target locations to save the energy. Moreover, UAVs tend to choose lower initial speed and lower acceleration speed in order to save energy when they face rather unstable wind ($\eta_A = 1e-2$) as shown by the blue curve in Fig.3b. In contrast, when the wind become more stable ($\eta_A = 1e-5$), higher initial speed and higher acceleration are more favorable as shown by the green curve in Fig. 3b.

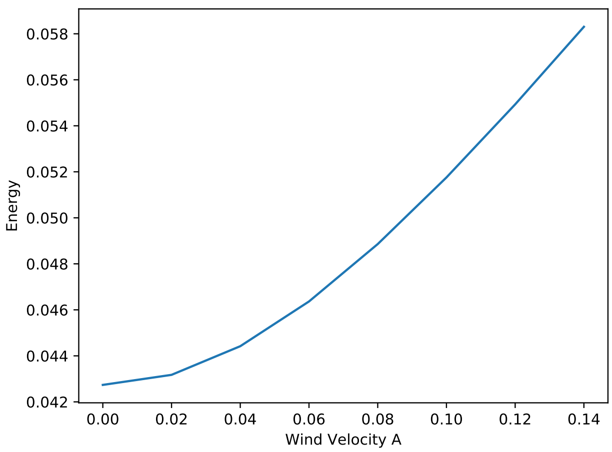


Fig. 4: Effect of wind velocity on energy consumption

V. Conclusion

In this paper, we consider the velocity control problem for massive rotary-wing UAVs in the metropolitan area after a disaster. In order to describe the frequent reconfiguration of massive UAVs, we formulate our problem as a Schrödinger bridge problem. Then we transform it into a mean field game problem in order to reduce the computation complexity. Then the G-Prox PDHG method is implemented to solve the mean field game problem due to its stability and faster speed to converge. In the simulation, we show the significant improvement of energy efficiency with our algorithm and analyze the effect of wind dynamics on the total energy consumption. In the future, we will further study on the general wind velocity field and a relaxed terminal density.

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