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ABSTRACT

Time dispersion plays an important role in the propagation of femtosecond pulses through water. The combined effects of time dispersion, radial diffraction and the Kerr nonlinearity on the pulse propagation are analyzed and it is shown that normal time dispersion leads to significant temporal broadening of ultrashort pulses and that it increases the threshold power for catastrophic self-focusing.

Keywords: time dispersion, ultrashort pulse, self focusing, optical breakdown

1 INTRODUCTION

The interpretation of high intensity laser experiments such as optical breakdown or retinal damage requires information on the electric field at the target area. In many cases a direct measurement is not possible and the electric field value is calculated using linear diffractive optics formulas for the pulse propagation through the media and the pulse characteristics before entering the media. The use of a linear model may be incorrect if the pulse power is comparable to the critical power for nonlinear self-focusing. In this work we suggest that in propagation of ultrashort pulses time dispersion also plays an important role. Therefore, there is a need for a simple model that predicts how the combination of radial diffraction, time dispersion and the Kerr nonlinearity effect the propagation of ultrashort laser pulses through an aqueous media.

Time dispersion can be normal or anomalous and its effect is different in each case. The qualitative picture is clearer in the case of anomalous time dispersion: Both time dispersion and radial diffraction are of the same sign and the propagation dynamics are determined by the balance between the 3D focusing Kerr nonlinearity and the 3D defocusing Laplacian. While the model that we use in this work cover this case, from now on we will concentrate on the more complicated case when normal time dispersion and radial diffraction have opposite signs.

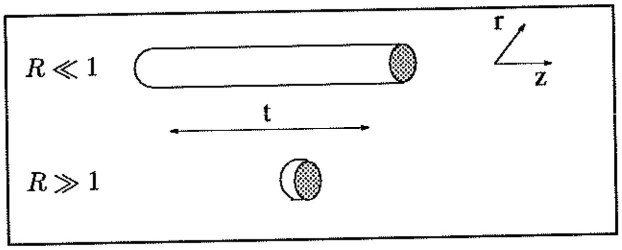

Figure 1: A cigar-like pulse (top) and a disc-like pulse (bottom)

In the absence of radial diffraction the equation governing the pulse propagation is the same as the equation for pulse propagation through optical fibers[3] and both normal time dispersion and the Kerr nonlinearity contribute to the temporal broadening of the pulse as it propagates. A different dynamics is observed in the absence of time dispersion: The pulse radius is determined by the competition between the self-focusing Kerr nonlinearity and radial diffraction. The magnitude of nonlinear self-focusing is proportional to the pulse power and is stronger than radial diffraction when the pulse power is above the critical power for self-focusing.

The main question that we address in this work is how the pulse propagates when all three mechanisms act together. Note that because of the nonlinearity one cannot simply superimpose the effect of each mechanism. Most theoretical studies have concentrated on the case of small time dispersion.[4,5] These studies showed that if the pulse is self-focusing even small normal time dispersion can have an important effect by delaying the onset of self-focusing and leading to the temporal splitting of the pulse into two pulses. The case of non-small normal time dispersion was studied in[2,6] and it was shown that normal time dispersion increases the threshold power for catastrophic self-focusing. Recently, Bergé and Rasmussen derived a mathematical model for propagation of Gaussian pulses under time dispersion, radial dispersion and Kerr nonlinearity.[1] In this work we use their model to analyze the interaction between these three mechanisms and to calculate the changes in the radial and temporal width of the pulse as it propagates.

2 THE MODEL

In this section we present a simplified model for ultrashort laser pulse propagation. For a derivation of the model see appendix 6.1.

The propagation of a femtosecond laser pulse through an aqueous media is described by the nonlinear Schrödinger equation with time dispersion (6). In this equation the relative magnitude of time dispersion can be written as[3]

$$\gamma = RG\ , \quad R = \left(\frac{r_0}{cT_0}\right)^2\ , \quad G = \omega_0 n_0(\omega)\frac{\partial^2}{\partial\omega^2}\Big[\omega n_0(\omega)\Big]_{\omega_0}\ . \tag{1}$$

The value of R is proportional to the ratio of the pulse initial radius r_0 to its initial temporal pulse width cT_0. Therefore, R is small for pulses that are 'cigar-like' (long and narrow) but is large for 'disc-like' (short and wide) pulses. Note that femtosecond pulses are usually 'disc-like': $R = 100$ for a 100 femtosecond pulse with 0.3 mm radius. G is the nondimensional group velocity dispersion of the media at the pulse frequency and its sign determines whether time dispersion is normal ($\gamma > 0$) or anomalous ($\gamma < 0$). In the visible regime $|G| \ll 1$ (since the value of n_0 for water is almost constant) and time dispersion magnitude can become comparable to radial dispersion only for disc-like pulses.

Direct analysis and simulations of the nonlinear Schrödinger equation with normal time dispersion are difficult. However, the mathematical model can be considerably simplified if we consider pulses with an initial Gaussian profile and assume that they maintain a Gaussian profile during their propagation. In this case the propagation dynamics can be approximated by[1]:

$$l_{zz} = \frac{1}{l^3(z)}\left[1 - \frac{P_0}{P_c}\frac{1}{L(z)}\right] \tag{2}$$

$$L_{zz} = \frac{1}{L^3(z)}\left[\gamma^2 + D\frac{P_0}{P_c}\frac{\gamma L(z)}{l^2(z)}\right], \quad D \cong 8.27 \ . \tag{3}$$

Here $L(z)$ and $l(z)$ are the temporal and radial broadening factors, respectively, z is given in units of the diffraction length $l_{diff} = k_0 r_0^2$, P_0 is the initial peak power and $P_c = 1.22^2\pi\lambda_0^2/(32n_0n_2)$ is the critical power for self-focusing in the absence of time dispersion. The initial conditions are given at the media interface at $z = 0$:

$$l(0) = 1 \ , \quad L(0) = 1 \ , \ l_z(0) = \frac{l_{diff}}{R_{lens}} \ , \quad L_z(0) = \gamma C \ . \tag{4}$$

and they include the possibility of an initial frequency chirp C and a lens with focal length R_{lens} located at $z = 0$.

3 THE PROPAGATION DYNAMICS

We begin by considering the case of an unchirped collimated beam i.e. $l_z(0) = L_z(0) = 0$. Equation (2) shows that the pulse focuses radially when its peak power $P_0/L(z)$ is larger than the critical power for self-focusing P_c and defocuses otherwise. Equation (3) shows that both normal time dispersion and the nonlinearity contribute to temporal broadening of the pulse. By solving

$$L_{zz} > \frac{\gamma^2}{L^3} \ , \quad L(0) = 1 \ , \quad L_z(0) = 0$$

we can get a lower bound estimate for temporal broadening:

$$L^2(z) > \gamma^2 z^2 + 1 \ .$$

Therefore, the pulse duration will more than double after it travels a distance of $2l_{diff}/\gamma$.

If the pulse power is below critical the propagation dynamics is determined by time dispersion and the nonlinearity. However, when $P_0 > P_c$ the picture is more complex. Let us denote by z_{TD} the point where in the absence of radial diffraction the peak pulse power becomes equal to P_c (i.e. $L(z_{TD}) = P_0/P_c$) and let z_{SF} be the blowup point in the absence of time dispersion where $l(z_{SF}) = 0$. There are two possible cases in (2)–(3) when $P_0 > P_c$:

- $z_{TD} < z_{SF}$

  In this case the power goes below critical before l reaches zero. Therefore, the pulse is undergoing a continuous temporal broadening while it focuses radially for $z < z_{TD}$ and defocuses radially afterwards.

- $z_{SF} < z_{TD}$

  In this case temporal broadening is not fast enough to arrest catastrophic self-focusing which will occur at z_{SF}.

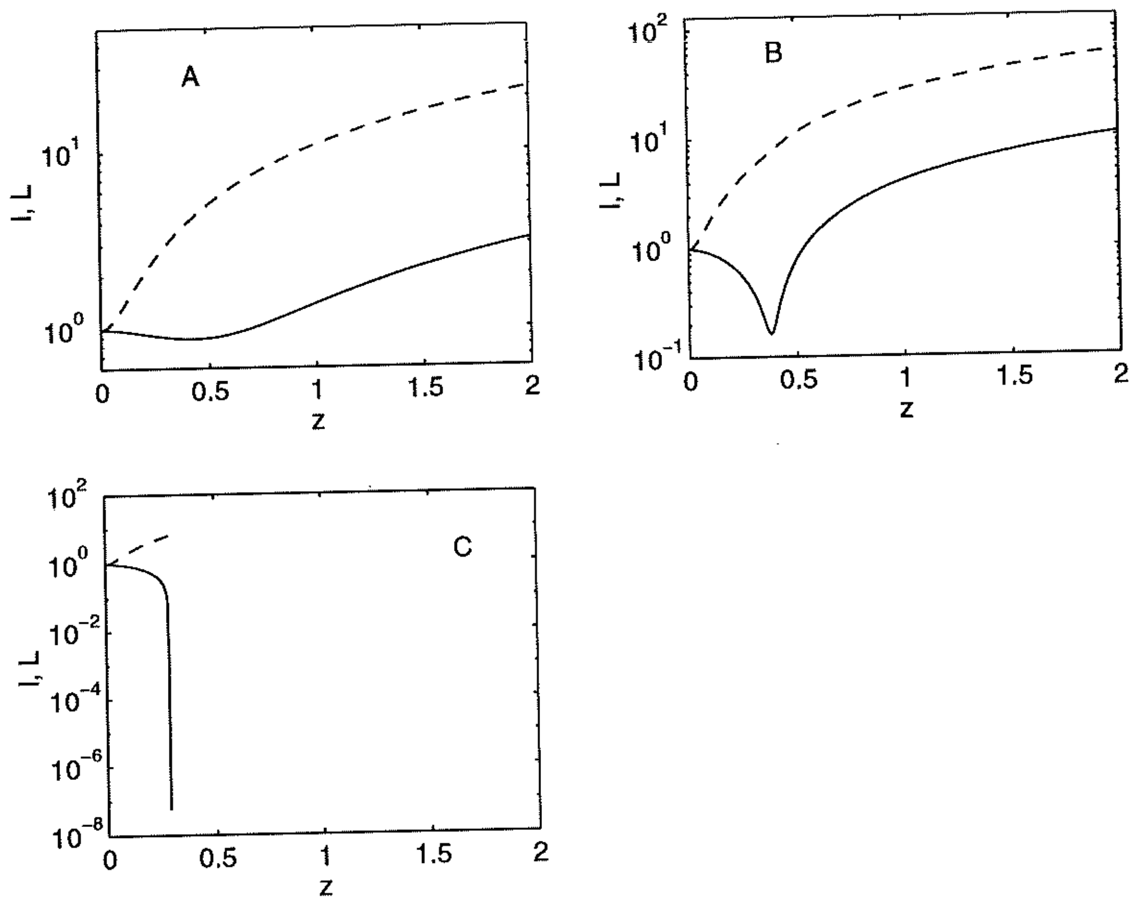

Figure 2: Temporal broadening factor L (dashed line) and radial broadening factor l (solid line) as a function of the distance of propagation z (in diffraction length units) for $\gamma = 1$ and a pulse power of A: $P_0 = 2P_c$, B: $P_0 = 6P_c$ and C: $P_0 = 8P_c$.

Figure 2 shows the evolution of $l(z)$ and $L(z)$ for an unchirped collimated pulse when $\gamma = 1$. Time dispersion clearly dominates in figure 2A and although the initial pulse power is twice the critical one for self-focusing P_c very little radial focusing takes place. Even when the input power is raised to $6P_c$ (figure 2B) time dispersion arrests self-focusing after focusing by a factor of ten. However, when the pulse power is $8P_c$ (figure 2C) nonlinearity dominates and catastrophic self-focusing occurs. In this case most radial focusing occurs very close to the blowup point.

The separation line between the two cases is

$$z_{TD} = z_{SF} \quad .$$

This equation can be solved for the threshold power for self-focusing P_{TH} as a function of γ and the initial conditions (appendix 6.2). Catastrophic self-focusing will occur if $P_0 > P_{TH}$ while temporal broadening will arrest radial focusing when $P_0 < P_{TH}$.

We are now in a position to analyze the effects of various parameters on the propagation:

- Effect of pulse duration:
  If the pulse is shorter, γ increases, temporal broadening is faster, z_{TD} is smaller and P_{TH} increases.

- Effect of initial pulse radius:
  A wider pulse corresponds to a larger value of γ and has the same effect in the model equations as that of a shorter pulse duration. However, the dynamics will occur over a larger physical distance since the diffraction length is longer.

- Effect of a focusing lens:
  A focusing lens is represented in the model by $l_z(0) < 0$. Therefore, radial focusing is faster, z_{SF} is smaller and P_{TH} decreases.

- Effect of a frequency chirp:
  A positive frequency chirp $C > 0$ corresponds to $L_z(0) > 0$. Therefore temporal broadening is faster, z_{TD} is smaller and P_{TH} increases. The opposite is true for $C < 0$.

- Effect of wavelength:
  The choice of wavelength effects the pulse propagation in many ways. Most importantly, it determines whether time dispersion is normal or anomalous. It also effects the size of γ and the length scale (l_{diff}) for changes in z.

4 DISCUSSION

The model provides qualitative information on the interaction between time dispersion, radial diffraction and the Kerr nonlinearity that was not available before. In addition, it allows experimentalists to estimate the pulse width, duration and power at the retina, near the location of optical breakdown etc. However, the quantitative predictions of the model are only correct to within an order of magnitude because of the error introduced by the assumption that the pulse maintains a Gaussian profile. For example, the results obtained for $P_{TH}(\gamma)$ using the model (appendix 6.2) are higher by a factor of two compared with the formula obtained by Luther et al[6] for the case of an unchirped collimated beam:

$$\gamma = \frac{1}{p}\left\{[3.38 + 5.5(p^2-1)]^{1/2} - 1.84\right\}[(p^{1/2} - 0.852)^2 - 0.0219]\,, \quad p = \frac{P_{TH}}{P_c}$$

The validity of the model breaks down when time dispersion is small and the pulse power is above critical since the Gaussian approach fails to capture the delicate balance between radial diffraction and the nonlinearity. Even if γ is not small, if the pulse power is greater than P_{TH} radial focusing will dominate over temporal broadening and the relative magnitude of time dispersion $\gamma(l/L)^2$ will become small near the blowup point. Therefore, while the model predicts catastrophic self-focusing ($l = 0$) when $P_0 > P_{TH}$, its validity breaks down once $l \ll L\gamma^{-1/2}$. Note that this only occurs near the blowup point where most of radial focusing takes place (see figure 2C). In this region modulation theory for small time dispersion becomes valid[4] and the pulse will split temporally into two pulses. Additional mechanisms can also become important in this region such as optical breakdown and beam nonparaxiality.

5 ACKNOWLEDGMENTS

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6 APPENDIX

6.1 Derivation of the model

The propagation of a laser pulse through an aqueous media is described by the nonlinear Schrödinger equation:

$$2ik\left(\frac{\partial\psi}{\partial z}+\frac{1}{v_g}\frac{\partial\psi}{\partial t}\right)+\frac{\partial^2\psi}{\partial r^2}+\frac{1}{r}\frac{\partial\psi}{\partial r}-k\frac{\partial^2 k}{\partial\omega^2}\frac{\partial^2\psi}{\partial t^2}+2k^2\frac{n_2}{n_0}|\psi|^2\psi=0 \ ,$$

where $\psi(z,r,t)$ is the envelope of the electric field, $r=\sqrt{x^2+y^2}$ is the radial coordinate, z is the axial coordinate in the direction of propagation, n_0 is the linear index of refraction of water, n_2 is the Kerr coefficient and $v_g=\partial\omega/\partial k$ is the group velocity. To bring this equation into a canonical form, we change into a retarded time frame $\tau=t-z/v_g$ and non-dimensionalize the variables:

$$\tilde{r}=\frac{r}{r_0}\ ,\quad \tilde{z}=\frac{z}{l_{diff}}\ ,\quad \tilde{\tau}=\frac{\tau}{T_0} \tag{5}$$

where r_0 is the pulse initial radius and $l_{diff}=k_0r_0^2$ is the diffraction length. The resulting equation is

$$i\frac{\partial\psi}{\partial\tilde{z}}+\underbrace{\frac{\partial^2\psi}{\partial\tilde{r}^2}+\frac{1}{\tilde{r}}\frac{\partial\psi}{\partial\tilde{r}}}_{\text{radial diffraction}}-\underbrace{\gamma\frac{\partial^2\psi}{\partial\tilde{\tau}^2}}_{\text{time dispersion}}+\underbrace{\kappa|\psi|^2\psi}_{\text{Kerr nonlinearity}}=0\ . \tag{6}$$

The relative magnitude of the nonlinearity is $\kappa|\psi|^2$, where

$$\kappa=2r_0^2k_0^2\frac{n_2}{n_0}$$

and that of time dispersion is

$$\gamma=\left(\frac{r_0}{T_0}\right)^2\left[k\frac{\partial^2 k}{\partial\omega^2}\right]_{\omega_0}\ .$$

which can be also written in the form (1).

We study the propagation of a pulse which enters the media at $z=0$ with an initial Gaussian profile and allow for the possibility that it has an initial frequency chirp C and that it is focused by a lens with focal distance R_{lens}:

$$\psi_0(r,t)=E_0\exp\left(-\frac{r^2}{2r_0^2}\right)\exp\left(-\frac{t^2}{2T_0^2}\right)\exp\left(\frac{ik_0r^2}{2R_{lens}}\right)\exp\left(-\frac{-iCt^2}{2T_0^2}\right)$$

or in dimensionless form:

$$\psi_0(\tilde{r},\tilde{\tau})=E_0\exp\left(-\frac{\tilde{r}^2}{2}\right)\exp\left(-\frac{\tilde{\tau}^2}{2}\right)\exp\left(\frac{il_{diff}\tilde{r}^2}{2R_{lens}}\right)\exp\left(-\frac{iC\tilde{\tau}^2}{2}\right)\ .$$

We drop the tilde signs and look for solutions of (6) that maintain a Gaussian profile:

$$\psi = \frac{E_0}{l(z)L^{1/2}(z)} \exp\left(-\frac{1}{2}\left(\frac{r}{l(z)}\right)^2\right) \exp\left(-\frac{1}{2}\left(\frac{\tau}{L(z)}\right)^2\right) \exp\left(i\left[\phi(z) - \frac{a(z)r^2}{4} + \frac{A(z)\tau^2}{4\gamma}\right]\right) \qquad (7)$$

Here $l(z)$ and $L(z)$ are the relative radial and temporal broadening, respectively. If we plug (7) into equation (6), the imaginary part of the equation is satisfied exactly if

$$a = -\frac{2l_z}{l}, \quad A = -\frac{2L_z}{L} \quad .$$

From the initial condition and the last relation we get the initial condition (4). The $O(1)$ terms in the real part of the equation determines $\phi(z)$:

$$\phi_z = -\frac{1}{l^2} + \frac{\gamma}{2L^2} + \frac{\kappa E_0^2}{2Ll^2} \quad .$$

The $O(r^2)$ and $O(t^2)$ equations are:

$$l_{zz} = \frac{1}{l^3}\left[1 - \frac{\kappa E_0^2}{L}\right] \qquad (8)$$

$$L_{zz} = \frac{1}{L^3}\left[\gamma^2 + \gamma\frac{\kappa E_0^2 L}{l^2}\right] \quad . \qquad (9)$$

Equations (8,9) were first derived by Bergé and Rasmussen.[1]

From the derivation we see that when we use the assumption that the pulse maintains a Gaussian profile (7) we neglect some $O(\tau^4, r^4)$ terms. Therefore, this assumption leads to results which are qualitatively correct but that do not give the correct quantitative answers. However, some improvement can be achieved by adjusting the constants in (8,9). For example, consider the case when there is no time dispersion ($\gamma = 0$). In this case $L \equiv 1$ and

$$l_{zz} = \frac{1}{l^3}\left[1 - 2\kappa N_0\right] \qquad (10)$$

where N_0 is the initial nondimensional peak power:

$$N_0 = \int |\psi_0(r,0)|^2\, r dr = \frac{1}{2}E_0^2 \quad .$$

Since we would like equation (10) to predict catastrophic self-focusing (i.e. l going to zero) when the peak (dimensional) power P_0 is greater than the critical one for self-focusing P_c, we replace it by

$$l_{zz} = \frac{1}{l^3}\left[1 - \frac{P_0}{P_c}\right] \quad . \qquad (11)$$

Similarly, in the absence of radial diffraction $l \equiv 1$ and

$$L_{zz} = \frac{1}{L^3}\left[\gamma^2 + 2\gamma\kappa N_0 L\right] \qquad (12)$$

We would like this equation to agree with the formula of Potasek et al[7] for temporal broadening in the absence of radial diffraction:

$$L^2(z) = 1 + 2^{-1/2}\pi\kappa\gamma E_0^2 z^2 + \gamma^2 z^2\left(1 + \frac{1}{3\sqrt{3}}\kappa^2\pi^2 E_0^4 z^2\right) \quad . \qquad (13)$$

From equation (13) (which was was derived for the case $L_z(0) = 0$):

$$L_{zz}(0) = \gamma^2 + \sqrt{2}\pi\kappa\gamma N_0 \quad .$$

Since

$$\frac{P_0}{P_c} = \frac{\kappa N_0}{N_c} \ , \quad N_c \cong 1.86 \ ,$$

we replace equation (12) with

$$L_{zz} = \frac{1}{L^3}\left[\gamma^2 + \gamma\frac{P_0}{P_c}L\right] \ , \quad D = \sqrt{2}\pi N_c \ . \tag{14}$$

Comparison of equations (8)–(9) with (11)–(14) leads to the model equations (2)–(3).

6.2 The threshold power for self-focusing $P_{TH}(\gamma)$

Let us define z_{SF} to be the blowup point (i.e. $l(z_{SF}) = 0$) in the absence of time dispersion. In that case l is changing according to (11) with the initial condition:

$$l(0) = 1 \ , \quad l'(0) = l'_0 \ . \tag{15}$$

The solution of this equation is

$$l^2(z) = c_1\left(z + \frac{l'_0}{c_1}\right)^2 + \frac{K}{c_1} \ , \quad K = 1 - \frac{P_0}{P_c} \ , \quad c_1 = (l'_0)^2 + K$$

Hence

$$z_{SF} = \frac{-l'_0 \pm \sqrt{P_0/P_c - 1}}{(l'_0)^2 + 1 - P_0/P_c} \tag{16}$$

and if $l'_0 = 0$ (collimated beam)

$$z_{SF} = \frac{1}{\sqrt{P_0/P_c - 1}} \ .$$

Similarly, define z_{TD} as the point where the maximum power goes below critical in the absence of radial diffraction. From (2), this will occur when $L(z_{TD}) = P_0/P_c$. To find z_{TD}, we solve equation (14) with

$$L(0) = 0 \ , \quad L'(0) = L'_0 \ .$$

Multiplying (14) by $2L_z$ and integrating, we have

$$L_z^2 = -\frac{\gamma^2}{L^2} - \frac{2D\gamma P_0/P_c}{L} + c \ , \quad c = \gamma^2 + 2D\gamma P_0/P_c + L_0'^2$$

Therefore,

$$\frac{dz}{dL} = \frac{L}{\sqrt{cL^2 - (2D\gamma P_0/P_c)L - \gamma^2}}$$

The solution of this equation subject to $L(0) = 1$ is

$$z = F(L) - F(1)$$

where

$$F(L) = \frac{1}{c}\sqrt{cL^2 - (2D\gamma P_0/P_c)L - \gamma^2} + \frac{D\gamma P_0/P_c}{c^{3/2}}\ln\left(2\sqrt{c}\sqrt{cL^2 - (2D\gamma P_0/P_c)L - \gamma^2} + 2cL - 2D\gamma P_0/P_c\right) \tag{17}$$

Therefore

$$z_{TD} = F\left(\frac{P_0}{P_c}\right) - F(1) \ . \tag{18}$$

The equation

$$z_{TD} = z_{SF}$$

can now be solved using (16)–(18) for P_{TH} as a function of γ and the initial conditions.

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