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Abstract. We study the asymptotic stability of a plane CJ detonation wave under the assumption of small resolved heat release (SRHR). We prove that the solution exists globally and that the solution converges uniformly to a shifted CJ detonation wave as $t \to +\infty$ for initial data which are small perturbations of the CJ detonation wave. The weighted energy method is used to overcome the difficulty arising from the sonic property at the end of the reaction. The SRHR model allows us to treat the non-monotone spike in the profile of the CJ detonation wave by the characteristic energy estimate.

Key words. CJ detonation, shock wave, traveling wave, sonic point, asymptotic behavior, weighted energy estimate, characteristic energy estimate.

AMS(MOS) subject classifications. 35L65, 35B40, 35B50, 76L05, 76J10.

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1 Introduction

We consider the combustion problem

$$u_t + (f(u) - q(1-\delta^2(1-z)))_x = \epsilon u_{xx} \tag{1}$$

$$z_x = k\varphi(u)z, \tag{2}$$

$\varphi(u)$ has the ignition form

$$\varphi(u) = \begin{cases} 0 & u \leq u_i \\ \text{smoothly increasing} & u_i < u < 2u_i \\ 1 & u \geq 2u_i \end{cases}$$

where $u_i > u_0 > 0$, $k, q, \delta, \epsilon > 0$ are constants and u and z are scalar functions of (x,t). f satisfies

$$f(0) = 0, f'(0) > 0, f''(u) > 0.$$

For a CJ detonation wave, there is a sonic point at the end of the reaction zone. That is, the speed of the fluid at the end of the reaction zone is sonic. This very property makes the late-time approach to the CJ detonation very slow [4]. This phenomena suggests us to consider the SRHR (small resolved heat release) model [1]. The model assumes two successive reactions. Most of the heat is released in the first reaction, which is assumed to be instantaneous. The rest of the heat, the 'resolved' fraction δ^2, is released in the second reaction, which is given a finite rate. The SRHR model reflects the important property that near the end of the reaction zone an $O(\delta^2)$ heat release is associated with a much larger $O(\delta)$ pressure drop [1]. Another motivation for choosing this model is that the detonation is stable to local 2D disturbances when the heat release of the detonation is small compared with the energy input to the flow by the supporting piston [3].

Under the assumption of SRHR, we are able to perform the characteristic energy estimate [9] [10] for the CJ detonation profile which is non-monotone due to the chemical reaction.

The weighted energy method [6] [13] is used to overcome the difficulty arising from the sonic property at the end of the reaction. The selection of the weight plays a crucial role. The weight depends on the rate of decay of the traveling wave profile at the far field.

We study the initial value problem (1) (2) with the following data:

$$u(x,0) = u_0(x) \tag{3}$$

$$z(+\infty,t) = 1 \tag{4}$$

where $u_0(x)$ is a small perturbation of the CJ detonation wave and satisfies certain conditions to be specified later.

We prove global existence of solution to the problem (1) (2) (3) (4) and that it's convergence to a shifted traveling wave solution.

In Section 2, we prove the results about CJ detonation wave solution including its decay rates at the far fields. Section 3 is the proof of the asymptotic stability of the CJ detonation wave. The proof consists of a construction of the weight function according to the rates found in Section 2 and the weighted energy estimate.

2 The CJ Detonation Wave Solutions

A CJ detonation wave solution is a solution of the following form

$$(u(x,t), z(x,t)) = (\psi(x - D_{CJ}t), Z(x - D_{CJ}t)) = (\psi(\xi), Z(\xi))$$

where $\xi = x - D_{CJ}t$ is the traveling wave variable and D_{CJ} is the speed of the CJ detonation wave. Then $(\psi, Z)(\xi)$ solves the following ordinary differential equations

$$-D_{CJ}\psi' + f'(\psi)\psi' = \epsilon\psi'' + q(1 - \delta^2(1 - Z))' \tag{5}$$

$$Z' = k\varphi(\psi)Z. \tag{6}$$

The boundary conditions are

$$\lim_{\xi\to-\infty} (\psi, Z)(\xi) = (u_l, 0) \tag{7}$$

$$\lim_{\xi\to+\infty} (\psi, Z)(\xi) = (u_0, 1). \tag{8}$$

The results about CJ detonation wave is the following.

Theorem 2.1 *There is a unique solution $(\psi,\ Z)$ to problem (5) (6) (7) (8). The propagating speed D_{CJ} satisfies*

$$D_{CJ} = \frac{f(u_l) - f(u_0) + q}{u_l - u_0}$$

and

$$D_{CJ} = f'(u_l). \tag{9}$$

Furthermore,

$$|\psi(\xi) - u_l| = O(\frac{1}{|\xi|}),\ \ \xi \to -\infty \tag{10}$$

and

$$|\psi(\xi) - u_0| = O(e^{-C|\xi|}), \ \ \xi \to +\infty \tag{11}$$

where $C > 0$ depends on ϵ and f.

Proof.

The proof of the existence and uniqueness of the profile can be found in Rosales and Majda [12] and the references therein. And so is the decay rate of the profile as $\xi \to +\infty$.

We now determine the rate of decay of the profile as $\xi \to -\infty$ where $f'(\psi(-\infty)) = D_{CJ}$. To do so, we prove the following lemma, with which the proof of the theorem is completed. ∎

Lemma 2.2

$$-\frac{1}{c_0\xi + c_1} + \frac{\delta\sqrt{qc}}{(c_0\xi + c_1)^2} < \psi(\xi) < -\frac{1}{c_0\xi + c_2}, \ \ \xi < \xi_3 < \xi_0 \tag{12}$$

for some constants c, c_0, c_1, c_2 and ξ_3.

Proof.

First, we have from (6) that

$$Z(\xi) = O(e^{k\xi}), \ \ \xi < \xi_3$$

for some $\xi_3 < \xi_0$.

Then plugging Z into the integrated (5), we have

$$-D_{CJ}(\psi(\xi) - u_l) + f(\psi(\xi)) - f(u_l) = \epsilon\psi'(\xi) + \delta^2 qce^{k\xi}, \ \ \xi < \xi_3 \tag{13}$$

where $c > 0$ is a constant.

Using the sonic property (9) of the CJ detonation profile around $\xi \to -\infty$, we rewrite the equation (13) in the following form:

$$\psi'(\xi) - c_0(\psi(\xi) - u_l)^2 = -\delta^2 q c e^{k\xi}$$

where $c_0 > 0$ depends on f and ϵ.

Let

$$y_1(\xi) = -\frac{1}{c_0\xi + c_1} + \frac{\delta\sqrt{qc}}{(c_0\xi + c_1)^2}$$

and

$$y_2(\xi) = -\frac{1}{c_0\xi + c_2}.$$

It is easy to check that y_2 and y_1 solve the following ODEs

$$y' - c_0 y^2 = -\frac{\delta^2 qc}{(c_0\xi + c_1)^4}$$

and

$$y' - c_0 y^2 = 0$$

respectively.

Since the source term $\delta^2 q c e^{k\xi}$ for $\psi(\xi) - u_l$ is in between those for y_2 and y_1, hence c_1 and c_2 can be chosen appropriately such that

$$y_2(\xi_3) < \psi(\xi_3) - u_l < y_1(\xi_3).$$

With the initial data ordered this way, we claim that

$$y_2(\xi) < \psi(\xi) - u_l < y_1(\xi), \quad \xi < \xi_3.$$

The claim can be proved by a maximum principle argument. ■

3 Asymptotic Stability of the CJ Detonation

Let

$$u(x,t) = \psi(x - D_{CJ}t) + v_x(x,t).$$

Then v, in traveling wave variable ξ, satisfies

$$v_t - D_{CJ}v_\xi + f(\psi + v_\xi) - f(\psi) - \delta^2 q(z - Z) = \epsilon v_{\xi\xi}.$$

It can be rewritten as

$$v_t + h'(\psi)v_\xi = \epsilon v_{\xi\xi} + F + \delta^2 q(z - Z) \tag{14}$$

where

$$h(u) = -D_{CJ}(u - u_l) + f(u) - f(u_l) \tag{15}$$

and

$$F = -(f(\psi + v_\xi) - f(\psi) - f'(\psi)v_\xi).$$

We make the following assumptions [4] [9]:

1. Small resolved heat release (SRHR):

   There are two successive reactions. Most of the heat is released in the first reaction, which is assumed to be instantaneous. The rest of the heat, the 'resolved' fraction $\delta^2 \ll 1$, is released in the second reaction, which is given a finite rate.

   So inside the reaction zone $\xi < \xi_0$,

$$0 < \int_{-\infty}^{\xi} f'(\psi(\xi))_\xi \, d\xi = \delta_1 < C\delta\sqrt{q}, \tag{16}$$

$$0 < f'(\psi(\xi))_\xi < \delta_2 < C\delta\sqrt{q}. \tag{17}$$

   This assumption makes the nonmonotone spike of the profile small so that the characteristic energy estimate can be obtained.

2. Assume *a priori* that

$$0 < \sup_{x,t} |v_x(x,t)| = \delta_3 \ll 1 \ . \tag{18}$$

This assumption will be guaranteed by the smallness of initial data and the stability analysis to be performed. The smallness condition (18) implies that there exist ξ_1 and ξ_2, $m > 0$ such that $\xi_0 < \xi_1 < \xi_2$ and

$$\phi(\psi) = \phi(u) = 0 \ , \qquad \xi > \xi_2 \tag{19}$$

$$\phi(\psi) = \phi(u) = 1 \ , \qquad \xi < \xi_1 \tag{20}$$

$$-f'(\psi(\xi))_\xi > m > 0 \qquad \xi_1 < \xi < \xi_2 \tag{21}$$

where ξ_0 is the maximum point on the CJ wave profile.

3. Zero initial integral difference:

$$\int_{-\infty}^{+\infty} (u_0(x) - \psi(x))dx = 0 \tag{22}$$

which implies that

$$v(\pm\infty, t) = 0 \ . \tag{23}$$

The reason for (23) to be true is that (1) makes the above integral $v(x,t)$ a conserved quantity, i.e.

$$\frac{d}{dt}\int_{-\infty}^{+\infty} (u(x,t) - \psi(x - D_{CJ}t))dx = 0,$$

or

$$\int_{-\infty}^{+\infty} (u(x,t) - \psi(x - D_{CJ}t))dx = \text{constant}.$$

Our main result is:

Theorem 3.1 *Suppose that* $v_0 \in H^2 \cap L^2_{<\xi>_-}$, $\|v_0\|_{H^2} + |v_0|_{<\xi>_-} \ll 1$ *and all the above assumptions hold, then the solution to (1) (2) (3) (4) exists globally and satisfies* $v(\cdot, t) \in H^2 \cap L^2_{<\xi>_-}$ *and*

$$\int_0^T (\|v(\cdot,t)\|_{H^2} + |v(\cdot,t)|_{<\xi>_-})dt \leq C(\|v_0(\cdot)\|_{H^2} + |v_0(\cdot)|_{<\xi>_-}), \tag{24}$$

for all $T > 0$ *and some constant* $C > 0$. *Consequently, we have*

$$\sup_{-\infty < x < +\infty} |u(x,t) - \psi(x - D_{CJ}t)| \to 0 \;\; as \;\; t \to +\infty, \tag{25}$$

where

$$< \xi >_- = \begin{cases} \sqrt{1+\xi^2} & \xi \leq 0 \\ 1 & \xi > 0. \end{cases}$$

Proof.

The sonic property of the CJ detonation wave causes one coefficient to vanish at the far field of the profile. This requires a nonregular perturbation treatment [4] which leads us to a weighted energy method [13] treatment. The weight depends on the rate of decay of the traveling wave profile at the far field.

Let us construct the weight function according to the decay rates (10) (11) of $\psi(\xi)$ at the far fields.

We choose the weight function around $\xi \to -\infty$ in the following way [13]

$$w(\xi) = w(\psi(\xi)) = \frac{(\psi - u_0)(\psi - u_l)}{h(\psi)} > 0, \; \xi < \xi_3. \tag{26}$$

As $\xi \to -\infty$, we have from the sonic property (9) of ψ at $-\infty$ and the definition of h (15) that

$$h(\psi) = O((\psi - u_l)^2).$$

Using the rate of decay of ψ at $-\infty$ (10), we have

$$w(\xi) = O(< \xi >).$$

For $\xi > \xi_3 + \delta_0$, we take $w(\xi) = 1$, where $\delta_0 > 0$ is a constant. Using an appropriate smooth function, which will be made clear below, to connect these two functions on $(\xi_3,\ \xi_3 + \delta_0)$, we finish the construction of the weight function w.

Now let us derive our basic weighted energy estimates.

Multiplying (14) by wv and integrating by parts, we get

$$\frac{d}{dt}\int_{-\infty}^{+\infty} \frac{1}{2} wv^2 dx + \int_{-\infty}^{+\infty} -\frac{1}{2}(wh)''(\psi)\psi_\xi v^2\ d\xi + \epsilon \int_{-\infty}^{+\infty} wv_\xi^2 d\xi$$

$$= \int_{-\infty}^{+\infty} (F + q\delta^2(z - Z))wv\ d\xi + \int_{-\infty}^{\xi_3+\delta_0} q\delta^2 Zw'(\psi)vv_\xi\ d\xi. \qquad (27)$$

Here we have used (5), the equation for the wave profile ψ, to have

$$h(\psi) = \epsilon\psi_\xi + q\delta^2 Z.$$

Use definition of w (26) to have

$$(wh)''(\psi) = 2 > 0,\ w = O(\frac{1}{\sqrt{-\psi_\xi}}),\ \xi < \xi_3 \qquad (28)$$

and

$$(wh)''(\psi) = h''(\psi) > 0,\ \xi > \xi_3 + \delta_0.$$

With a proper choice of w on $(\xi_3,\ \xi_3 + \delta_0)$, so is the sign of $(wh)''(\psi)$ for $\xi_3 < \xi < \xi_3 + \delta_0$. Here $w'(\psi) = \dfrac{w'(\xi)}{\psi_\xi}$ on $(\xi_3,\ \xi_3 + \delta_0)$.

Noticing that $Z(\xi)$ decays exponentially as $\xi \to -\infty$, by taking $|\xi_3|$ large enough and δ small enough, the last term on the right hand side of (27) can be absorbed into the two terms on the left hand side.

$$\int_{-\infty}^{\xi_3+\delta_0} q\delta^2 Zw'(\psi)vv_\xi\ d\xi \le C\delta(\int_{-\infty}^{\xi_3+\delta_0} -\frac{1}{2}(wh)''(\psi)\psi_\xi v^2\ d\xi + \epsilon\int_{-\infty}^{\xi_3+\delta_0} wv_\xi^2 d\xi),$$

where we have used the Schwarz inequality.

Since $\psi_\xi > 0$ for $\xi < \xi_0$ and $\psi_\xi < 0$ for $\xi > \xi_0$, we do the estimates separately.

In the region $\xi > \xi_0$, the sign of ψ_ξ is a good one. While in the region $\xi < \xi_0$, we use the characteristic energy estimate [9] [10] and the SRHR assumption to control the increasing part. The underlying reason that we could do this is because the SRHR assumption [4] assumes that the increasing part (resolving part of the energy) is much smaller than the decreasing part (the heat released instantanously at the front) of the CJ detonation profile.

Now we use the characteristic energy method [9] to estimate the increasing part of the second term on the left hand side of (27). The idea is to integrate the weighted equation for v along characteristic direction to get v^2 and then plugging it into the integration. The key conditions here are that $|\psi(\xi) - \psi(-\infty)|$ and $|\psi_\xi|$ are bounded by $\delta\sqrt{q}$, see (16) and (17).

Let

$$S(\xi) = S(\psi(\xi)) = \frac{1}{h'(\psi(\xi))w(\psi(\xi))}.$$

Then from (10), (15) and (26) we have that

$$S(\psi(\xi)) = O(1), \quad S'(\psi(\xi)) = O(\frac{1}{\psi(\xi) - u_l})\ \xi \to -\infty. \tag{29}$$

Multiplying (14) by wvS, and then integrating from $-\infty$ to ξ, we have

$$\begin{aligned}
\frac{1}{2}v^2(\xi,t) &= \int_{-\infty}^{\xi} S(\eta)(-wvv_t + \epsilon wvv_{\eta\eta} + q\delta^2\ (z-Z)\ w\ v + Fwv)d\eta \\
&= \int_{-\infty}^{\xi} S(\eta)(-wvv_t + q\delta^2(z-Z)wv + Fwv)d\eta + \\
&\epsilon\int_{-\infty}^{\xi} S(\eta)(-wv_\eta^2)d\eta + \epsilon S(\xi)(wvv_\xi) + \epsilon\int_{-\infty}^{\xi} S'(\psi)\psi_\eta(-wvv_\eta)d\eta \\
&= \int_{-\infty}^{\xi} S(\eta)(-wvv_t + q\delta^2 w(z-Z)v + Fwv - \epsilon wv_\eta^2)d\eta \\
&+\epsilon S(\xi)wvv_\xi + \epsilon\int_{-\infty}^{\xi} S'(\psi)\psi_\eta(-wvv_\eta)d\eta.
\end{aligned}$$

Now multiplying the above inequality by $\psi(\xi)_\xi$ and integrating from $-\infty$ to ξ_0, using Schwarz's inequality and Fubini's theorem, we have

$$
\begin{aligned}
&\int_{-\infty}^{\xi_0} \frac{1}{2} v^2(\xi,t)\psi(\xi)_\xi \, d\xi \\
&= \int_{-\infty}^{\xi_0} \psi(\xi)_\xi \, \epsilon S(\xi) w(\xi) v \; v_\xi \, d\xi \\
&+ \int_{-\infty}^{\xi_0} \int_{-\infty}^{\xi} \psi(\xi)_\xi S(\eta) w(\eta)(-v v_t + q\delta^2(z-Z)v + Fv - \epsilon v_\eta^2) d\eta \; d\xi \\
&+ \int_{-\infty}^{\xi_0} \int_{-\infty}^{\xi} \psi(\xi)_\xi S'(\psi(\eta))\psi(\eta)_\eta w(\eta)(-\epsilon v v_\eta) d\eta \; d\xi \\
&\le \frac{1}{8} \int_{-\infty}^{\xi_0} \psi(\xi)_\xi \frac{v^2}{2} \, d\xi + C\delta_2 \epsilon \int_{-\infty}^{\xi_0} w(\xi) v_\xi^2 \, d\xi \\
&+ \int_{-\infty}^{\xi_0} \int_{\eta}^{\xi_0} \psi(\xi)_\xi S(\eta) w(\eta)(-v v_t + q\delta^2(z-Z)v + Fv) d\xi \; d\eta \\
&+ \int_{-\infty}^{\xi_0} \int_{\eta}^{\xi_0} \psi(\xi)_\xi S'(\psi(\eta))\psi(\eta)_\eta w(\eta)(-\epsilon v v_\eta) d\xi \; d\eta .
\end{aligned}
$$

We have used

$$\psi_\xi(\xi) w(\xi) = O(\frac{1}{|\xi|}), \;\; \xi \to -\infty,$$

which can be made small by choosing $|\xi_3|$ large enough.

Further use of the above equality, (9), (16), (17), (29) and Schwarz's inequality leads us to

$$
\begin{aligned}
&\int_{-\infty}^{\xi_0} \frac{1}{2} v^2(\xi,t)\psi(\xi)_\xi \, d\xi \\
&\le \frac{1}{4} \int_{-\infty}^{\xi_0} \frac{1}{2} v^2(\xi,t)\psi(\xi)_\xi d\xi + C(\delta_1+\delta_2)\epsilon \int_{-\infty}^{\xi_0} w(\xi) v_\xi^2 \, d\xi \\
&+ C\delta_1 \frac{d}{dt} \int_{-\infty}^{\xi_0} w(\eta)(-\frac{1}{2} v^2(\eta,t)) d\eta + C\delta_1 \int_{-\infty}^{\xi_0} w(\eta)(q\delta^2(z-Z)v + Fv) \; d\eta .
\end{aligned}
$$

Solving for $\int_{-\infty}^{\xi_0} \frac{1}{2} v^2(\xi,t)\psi(\xi)_\xi \, d\xi$ and plugging it back into (27), we have

$$
\begin{aligned}
&\frac{d}{dt} \int_{-\infty}^{+\infty} \frac{1}{4} w v^2 dx + \int_{-\infty}^{+\infty} \frac{1}{2} (wh)''(\psi)|\psi_\xi| v^2 \, d\xi + \frac{\epsilon}{2} \int_{-\infty}^{+\infty} w v_\xi^2 d\xi \\
&\le C \int_{-\infty}^{+\infty} (F + q\delta^2(z-Z)) w v \, d\xi. \qquad (30)
\end{aligned}
$$

Now noticing that δ can be made small by the SRHR assumption, we estimate $\int_{-\infty}^{+\infty} q\delta^2(z-Z)wv\ d\xi$ in the same way as in [9].

Smallness assumption (18) allows us to treat the higher order term F.

We conclude at the following main estimate

$$\frac{d}{dt}\int_{-\infty}^{+\infty}\frac{1}{2}wv^2dx+\int_{-\infty}^{+\infty}\frac{1}{2}(wh)''(\psi)|\psi_\xi|v^2\ d\xi+\epsilon\int_{-\infty}^{+\infty}wv_\xi^2d\xi\leq 0.$$

Or

$$\int_{-\infty}^{+\infty}\frac{1}{2}wv^2dx+\int_0^t\int_{-\infty}^{+\infty}\frac{1}{2}(wh)''(\psi)|\psi_\xi|v^2\ d\xi+\epsilon\int_0^t\int_{-\infty}^{+\infty}wv_\xi^2d\xi\leq C|v_0|_w^2.$$

Similarly, we have the following estimates

$$\int_{-\infty}^{+\infty}\frac{1}{2}v_\xi^2dx+\epsilon\int_0^t\int_{-\infty}^{+\infty}v_{\xi\xi}^2d\xi\leq C(|v_0|_w^2+|v_{0\xi}|^2)$$

and

$$\int_{-\infty}^{+\infty}\frac{1}{2}v_{\xi\xi}^2dx+\epsilon\int_0^t\int_{-\infty}^{+\infty}v_{\xi\xi\xi}^2d\xi\leq C(|v_0|_w^2+\|v_{0\xi}\|_1^2)$$

for v_ξ and $v_{\xi\xi}$ respectively. Here the main estimate for v has been used to get the above estimate for v_ξ. So is for $v_{\xi\xi}$.

Hence

$$\begin{aligned}&|u(x,t)-\psi(x-D_{CJ}t)|=|v_x(x,t)|\\&=(2\int_{-\infty}^{x}v_xv_{xx}(y,t)dy)^{\frac{1}{2}}\\&\leq(\int_{-\infty}^{+\infty}v_x^2(x,t)dx+\int_{-\infty}^{+\infty}v_{xx}^2(x,t)dx)^{\frac{1}{2}}\\&\leq\|v(\cdot,t)\|_2\to 0,\ t\to+\infty.\end{aligned}$$

That finishes the proof of the Theorem.

■

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