UCLA
COMPUTATIONAL AND APPLIED MATHEMATICS

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February 1997

CAM Report 97-4

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1 Hierarchical Structures and Scalings in Turbulence

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Abstract. Turbulence is a mixture of hierarchical structures (eddies) of different sizes, different amplitudes and different degree of coherence. The size distributes continuously from the integral scale ℓ_0 to dissipation scale η. At each scale ℓ ($\ell_0 < \ell < \eta$), the most intermittent structures have the highest amplitude with the highest degree of coherence. Structures of lower amplitudes are related to the most intermittent structures according to a symmetry relation which defines the hierarchy. This is the Hierarchical Structure model (She & Leveque, 1994, *Phys. Rev. Lett.*, **73**, 211) which links fluctuation structures of various sizes and amplitude all together.

Moments of the velocity increments across distance ℓ vary in power law with ℓ in the inertial range. The whole set of scaling exponents ζ_p of the p-th order moments carry a rich set of statistical information about the fully developed turbulent field. It is now widely recognized that ζ_p vary nonlinearly with p, which is often described as the multi-scaling or anomalous scaling problem. The Hierarchical Structure model describes ζ_p in teams of parameters which characterize the most intermittent structures. In isotropic turbulence, they are conjectured to be filaments, and the resulting expression for ζ_p reads $p/9 + 2(1 - (2/3)^{p/3})$.

The Hierarchical Structure model can be derived by a random cascade model of the log-Poisson type. A more elegant derivation calls for an invariance principle and the plausible assumptions that eddies have no characteristic size other than ℓ_0 and no characteristic amplitude other than that of the most intermittent structures. The experimental validation of the model and its implications are discussed in great detail. It is concluded that the description of turbulence in terms of hierarchical structures is physically sound and promising.

1 Turbulence: An Old Problem

Turbulence is a subject of long history. Over the past century, a huge amount of experimental and theoretical investigation has been devoted to its study, yet general consensus have not been reached upon as what is *the solution* of turbulence. It is perhaps naive to try to look for *the solution* of turbulence, it is nevertheless possible to identify a few basic features which the community generally agree that turbulence possesses. First, turbulence (velocity, temperature, tec.) is a highly irregular vector field with excitations spreading *continuously* over many scales both in space and time. The wide range of excited scales in both space and time imply the existence of a tremendous amount of disorders and complexity, or, in theoretical teams, of a huge number of degrees of freedom. Second, strong nonlinear interaction is primarily responsible for spreading excitations over many scales. Consequently, turbulent fluctuations, however random they may be, contains certain structures of various degree of coherence. Experimentalists and computing scientists have carried out measurements of various physical quantities, struggling to define this mixed correlation. Theoreticiens have developed many mathematical tools (singular perturbatioon, renormalized perturbation, etc.), fighting for describing such a mixture of orders and disorders. Although the success has been limited, the impact of a breakthrough will undoutedly go beyond the area of fluid mechanics; it will increase our analytical ability to characterize the complexity generated by nonlinear processes in general.

1.1 Statistical Description of Turbulence

In the development of the theory of turbulence, several major trends may be noticed. Early in 20th century, turbulence was characterized by the second order correlation functions whose Fourier transform is related to the energy spectrum. It is a natural tool to use to describe a random field which was strongly inspired by the development of the statistical mechanics and of the theory of random functions. Indeed, it gives the first approximation to a near Gaussian stochastic field which carries minimal internal correlation, as turbulence appears to be (see further discussions later). Work in this direction has led to the paradigm now described as the statistical description of turbulence. While many pioneers have contributed to the development of the ideas and tools, the 1941 work of Kolmogorov (1941) has made a particular impact. The reason seems to be that Kolmogorov (1941) made a specific prediction about the energy spectrum of turbulence at very large Reynolds number, which, whether right or wrong, has make one more seriously consider some ideas such as the local cascade and the dimensional argument. These ideas have since been instrumental in the study of the dynamics of turbulence in many areas, specially in geophysical context.

Starting in late fifties, there have been important developments in the analytical description of turbulence stimulated by the progress in the quantum field theory. The pioneer work of Kraichnan (1959) and many others aimed at rationalize the statistical description of turbulence with field-theoretical analysis of stochastic fields constrained by the dynamics of the Navier-Stokes equation (for more discussions, see Proccacia & L'vov in this volume). Difficulties appeared immediately because an infinite series resummation is necessary. Closure assumption must be introduced to regulate this resummation of divergent series, and it is now realized that except for a few model problems, the regulated problems could be arbitrarily different from the original problem. There is no known regulatory procedure (including the renormalization group analysis) which is free of uncontrolled deviation from the original problem. There is one common feature among most, if not all, formal regulatory (closure) procedures, that is, the statistics of the velocity fluctuations are close to Gaussian uniformly at all scales. In other words, the description of turbulence by closure theories seem to assume minimal structures in turbulence, or in Kraichnan's words (1959), with maximum stochasticity.

Recently, there arises new attempts of non-perturbative closure theory for the statistics of a passive scalar advected by a white-noise velocity field (Kraichnan 1994). See also discussions of L'vov and Proccacia in this volume.

1.2 Structural Description of Turbulence

It has long been the goal of experimental fluid dynamics community to characterize turbulence in terms of certain basic fluid structures distributed randomly in space and time. There, one seeks the "atoms" that are the fundamental constituents of turbulence. In seventies and eighties, hopes arise because of the discovery of the so-called "coherent structures" which were thought to control the fundamental processes of turbulence. While the studies have led to detailed knowledge about certain complicated fluid mechanical processes, it is also realized that there are a big variety of the "coherent structures" which are geometrically too complicated to describe by available mathematical tools, and which are too varied to be regarded basic. Nevertheless, there is a general consensus that at the location of very strong vorticity or of very low pressure, the filamentary structures appear. Both laboratory experiments and numerical simulations have confirmed the existence of the so-called vortex filaments (Douady *et al.* 1991, Siggia 1981, She *et al.* 1990, Vincent & Meneguzzi 1991, etc.). The degree of coherence in these regions are relatively greater, compared to other regions of lower vorticity amplitude where structures appear to have much more complex forms. It is fair to say from various observations that turbulence does not contain one kind of "basic" structures, but rather a series of complicated structures.

Note that the attempt to describe turbulence as a superposition of some basic structures (patterns) has an important shortcoming. Since the fully developed turbulence is strongly nonlinear at all scales, except at scales smaller than the viscous dissipation cutoff, the existence of any basic structure demands extraordinary condition such that the structure is held against strong disturbance from the mutual interaction with others. We know one example, the Burgers' shock structure which exists because of the infinite compressibility which make the dynamics local and stable. The three-dimensional incompressible Navier-Stokes equations describe however just an oppoite situation of very nonlocal and unstable dynamics, where there is no evidence that any entity holds itself stably. Rather, an quasi-equilibrium holds in a statistical sense with active dynamics of the structures at all scales: stretching, deformation, folding, reconnection, etc. The consequence is that turbulent structures of many kinds co-exist dynamically. Turbulence is unlikely to be a family of one animal, but likely to be a zoo!

2 Scaling of Turbulence: Quantitative Studies

There is little disagreement that turbulence involves a great deal of complexity and it is generally agreed that its properties are only stable and measurable in a statistical context. The problem we are facing is what is the reliable statistical information, before we start to consider how the information to be used effectively in practical modeling. The study of turbulence in the past has placed overwhelming emphasis on the later issue which is understandably driven from a practical standpoint. In late eighties and nineties, there arise a new trend in the studies of turbulence with the participation of experimental physicists. The "new" Experiments are carefully controlled, the measurements are better calibrated, and more importantly the studies were more physically motivated (e.g. Tong *et al.* 1988, Benzi *et al.* 1994, Noullez *et al.* 1996, Tabeling *et al.* 1996; see also Sreenivason & Antonia 1996). These studies have led to many accurate measurements, and have provided interesting reliable quantitative outputs. One of the measured quantities of great interest is the scaling exponent.

2.1 Physical Significances of Scaling

The scaling behavior is one of the most intriguing aspects of fully developed turbulence. It refers to the observation that in high Reynolds number flows, moments of the velocity difference across a distance ℓ (the so-called velocity structure function) varies in power law as ℓ, or moments of the energy dissipation varies with the Reynolds number (R_λ) (in leading order) in power law. The scaling exponents characterize how fast the moments decrease as $\ell \to 0$, or increase as $R_\lambda \to \infty$. If the exponents are known, then it is possible to predict the moments of the velocity fluctuations at any (smaller) scale based on large scale data, or the moments of the dissipation fluctuations at any (larger) Reynolds number based on moderate Reynolds number data. This is one aspect of the practical interests.

The theoretical interests are more intriguing. Compared to other statistical quantities such as moments themselves or the probability distribution functions (PDF), the scaling exponents only address the relative changes with ℓ or with R_λ, which can be more universal. Kolmogorov (1941) in fact conjectured that the scaling exponents are universal, independent of the statistics of large-scale fluctuations, the mechanism of the viscous damping and the flow environment, when the Reynolds number is sufficiently large. There exist a number of experimental measurements in homogeneous open turbulent flows, i.e. "free" turbulence far from the boundaries (Anselmet *et al.* 1984, Benzi *et al.* 1994a), which show evidence of the existence of such universal scaling laws. More interestingly, the experimentally measured values also agree quantitatively with those measured in computer-simulated isotropic Navier-Stokes turbulence with a simplistic boundary conditions, i.e. periodic boundary conditions.

Clearly, a set of dynamical information is contained in the values of the scaling exponents. The study of the scaling laws attracts a great deal of attention in physics in the study of critical phenomena. Here, turbulent medium shows a few degree more complicated than the equilibrium statistical mechanical systems of phase transition. Theoretically speaking, turbulence is a nonequilibrium medium with cascade of the energy flux from large to small scales. Observationally, this difference is reflected by a nonlinear dependence of the scaling exponents on the order of the velocity structure functions, the so-called multi-scaling. In particular, the measured scaling exponents deviate from the Kolmogorov 1941 (K41) theory which predicts a linear dependence. Note that the K41 model, until recently, has been the only predictive model of scaling laws with no adjustable parameter. This anomalous scaling problem has attracted much attention, because it is believed that the rich information contained in the entire serie of exponents provide important hint about the self-organization of turbulent structures. Furthermore, only the accurate quantitative results can provide the necessary ground for testing various theoretical descriptions.

2.2 Extended Self-Similarity

One of the important developments in the measurement of scaling exponents is a work of Benzi *et al.* (1993) about the Extended Self-Similarity (ESS) property of turbulence. It is discovered by Benzi *et al.* that the velocity structure function of any order p (reasonably accurate with a sufficient sample size) depends on the structure function of order q (usually chosen to be 3) in a much better power law. In particular, when the structure functions deviate from the power-law behavior as the scale is approaching to the viscous cutoff, the relative power law behavior of p-th moment versus q-th moment holds still remarkably well until at scales very close to the Kolmogorov dissipation cutoff. A number of studies were reported after the initial discovery (see more references in Benzi *et al.* 1996b), and indicated that the ESS property works with varied efficiency in different physical environments, but holds real well in far-field homogeneous flow. These further studies confirm the existence of the ESS phenomenon, and provide important information to a better understanding of the physical origin of ESS. Since the third order structure function is expected to be linearly proportional to the length scale in the inertial range, the relative scaling exponent of the p-th order structure function with respect to the third order is taken to be a new method of measuring ζ_p which show improved reliability.

In mathematical team, the ESS property indicates that as the viscous range is approached, the velocity structure functions of different orders show the same characteristic deviations from a power law behavior in such a way that their relative functional dependence is preserved. More physically speaking, there is one characteristic quantity which controls the deviation of the whole set of the velocity structure function from the inertial range scaling behavior. The word "extended" indicates possibly the existence of other self-similarity property of turbulence which persists even when this quantity shows its non-power law dependence on the length scale. We will offer more discussions below which support this conclusion.

An even more interesting development is some later work by the same group (Benzi *et al.* 1996a-b) which have reported the experimental evidence of a Generalized ESS property (GESS) satisfied by flows with a variety of physical conditions where the ESS property is not satisfied. Instead of studying the velocity structure functions which are moments, they propose to study the normalized structure functions with respect to a certain order which are generally referred to as the hyper-flatness factors. In the same way, they suggest to evaluate the relative functional dependence of those hyper-flatness (HF) factors. The result is that the HF factors are in a beautiful power-law with each other through the w hole range of length scale explored (from the intergral scale down to very small scale) and for a variety of different flow conditions such as with or without a shear, near a boundary layer, having relatively small Reynolds number, where the ESS property is known to not work.

The interest of these work is two-fold. First, it leads to a better way to estimate the scaling exponent. The measured scaling exponents do not have the same meaning, strictly speaking, as the original ones proposed by Kolmogorov, the so-called inertial range. But Benzi *et al.* (1994, 1995) have shown that when the Reynolds number is large enough to show a section of the inertial range in the traditional sense, the exponents estimated by ESS or GESS at smaller Reynolds number are consistent with the estimate of the true inertial range exponents. Therefore, the ESS or GESS provides us a more accurate way for measuring the inertial range exponents. Second, it points out a more fundamental scaling property of turbulence. This will become clear only if we provide some real physical understanding of the phenomenon. This is the central topic of our discussion below.

3 Models of Anamolous Scalings

During the past thirty years, many theoretical approaches have been suggested to address the anomalous scaling behavior of turbulence. Many scaling models start with a very specific ansatz for the PDF of the coarse-grained energy dissipation at the inertial-range scales. The most famous one is the log-normal model (Kolmogorov 1962). These models violate, in one way or another, certain exact inequality (Novikov 1971) for the scalings of high order moments, and therefore, can be considered at best as some approximations but not an overall good prescription of the inertial-range statistics. A more widely accepted approach is built upon the notion of multifractality of turbulence (Mandelbrot 1974, Parisi & Frisch 1985, Meneveau & Sreenivasan 1987). Statistically, it describes the inertial-range cascade as a (discrete) random multiplicative (RM) process; the probability distribution of the corresponding RM coefficient $\mathcal{W}$, $P(\mathcal{W})$, fully determines the inertial-range scaling exponents ζ_p. Since $P(\mathcal{W})$ can be described in terms of arbitrarily many parameters, the resulting scaling formula may exhibits *a priori* any concave nonlinear dependence on the order p (Parisi & Frisch 1985). The problem arising in this approach is therefore the arbitrariness of the model; in other words, the physical, or fluid mechanical meaning of the RM process appears very obscure. Consequently, the parameters in the ansatz $P(\mathcal{W})$ remain purely adjustable parameters.

There have long been approaches which attempted to understand the scaling from a more physical or mathematical basis, e.g., the work of Tennekes (1968), Lundgren (1982), Chorin (1991, 1992), Gilbert (1993), Pullin & Saffman (1993), Saffman and Pullin (1994), among others. Fluid structures which are local solution of the Navier-Stokes equations are randomly superposed in some way for computing the statistical correlations. There have been many predictions of the energy spectrum, but the scalings of high order correlations are difficult to calculate technically. Moreover, we believe that any local solution cannot encompass the whole complexity of the turbulence statistics, because the strong nonlinearity contradicts the fundamental linear superposition principle. It is likely that long-range correlations of a whole set of local solutions play a dominant role in determining the global state of turbulence.

4 Hierarchical Structure Model of Turbulence

In what follows, we will describe a relatively new approach (She & Leveque, 1994) which has shown features of both the structural approach and the random cascade approach. Based on an assumption of a symmetry, the model predicts the scaling exponents in terms of the properties of the most intermittent structures. The later correspond to observable fluid mechanical features of considerable coherence embedded in a disordered turbulent medium. This model, called Hierarchical Structural Model, acknowledges the overwhelming complexity of fully developed turbulence (except the most excited, intermittent structures), but point out a novel simplicity which is the symmetry across length scale and across the amplitude of fluctuations. It was shown (Dubrulle 1994, She & Waymire 1995) that the symmetry is exactly realized by a RM process of a log-Poisson type (and thus also called log-Poisson model).

4.1 Physical Picture

A cartoon picture of turbulence from the viewpoint of the Hierarchical Structure model (She & Leveque 1994) can be described as follows. When turbulence are excited in a three-dimensional domain at the so-called integral scale, the nonlinear interaction spreads the fluctuations to small scales. The dynamical turnover time decreases with scale, so do the root-mean-square (rms) velocity fluctuations. However, with respect to the rms velocity fluctuations, there develop increasingly rare and large amplitude events as the cascade proceeds. Higher are the amplitude of the fluctuations, more coherent are they in spatial configurations, and more phase correlated across length scales. At the statistically steady state, the fluctuations at large and small scales and at large and small amplitudes form a unified hierarchy described by a certain symmetry.

The probability density function (PDF) of velocity fluctuations at the integral scale reflects the motions directly excited by an external mechanism, and thus is not universal. However, the PDFs at smaller scales can be described by a transformation defined by the symmetry which will be a convolution with the integral-scale PDF. It is believed that the transformation (and the symmetry) is intrinsic to the nonlinear dynamics, and can be determined by universal physical principles.

4.2 The Model

In searching to define this transformation, She & Leveque (1994) proposed to study a hierarchy defined through the ratio of the successive moments: $\epsilon_\ell^{(p)} = \langle\epsilon_\ell^{p+1}\rangle/\langle\epsilon_\ell^p\rangle$ $(p = 0, 1, ...)$, where ϵ_ℓ is the coarse-grained energy dissipation at an inertial-range scale ℓ. This hierarchy passes from the mean field described by $\epsilon_\ell^{(0)}$ and the most intermittent structures described by $\epsilon_\ell^{(\infty)} < \infty$ (the upper bound for the field ϵ_ℓ in a finite space-time manifold). While p can be any real number, restricting to the set of integers make the presentation easy to follow. Since the pth order ratio can have a nontrivial scaling: $\epsilon_\ell^{(p)} \sim \ell^{\lambda_p}$, turbulence will generally behave as a multiscaling field. It is interesting to note that $\epsilon_\ell^{(p)}$ represents a sequence of dissipation events with increasing amplitudes when the underlying PDF of ϵ_ℓ exhibited a log-concave tail which is quite true from experimental observations. Therefore, λ_p's describe scaling properties of structures of various amplitude in the physical space.

When $\lambda_0 = \lambda_\infty$, the scaling exponents of all members of the whole hierarchy is identical. The scaling field is said to be a monoscaling field which is statistically equivalent to a fractional Brownian motion. In this case, the scaling exponents τ_p, defined by $\langle \epsilon_\ell^p \rangle \sim \ell^{\tau_p}$, depend linearly on p. The K41 can be recovered as a special case ($\lambda_\infty = \lambda_0 = 0$). Otherwise, it leads to the β-model (Frisch, Nelkin & Sulem 1978). When $\lambda_0 \neq \lambda_\infty$, we have generically a multifractal field. Intuitively, there should be a relation among λ_p's, since the whole hierarchy is the result of a unique dynamics (the Navier-Stokes dynamics) in which low-order and high-order moments are consistently related. She and Leveque (1994) further proposed that there exists a symmetry, that is,

$$\epsilon_\ell^{(p+1)} \sim \epsilon_\ell^{(p)^\beta} \epsilon_\ell^{(\infty)^{(1-\beta)}}; \qquad \lambda_{p+1} = \beta \lambda_p + (1-\beta)\lambda_\infty, \tag{4.1}$$

where β is a constant independent of p. Under (4.1), the scaling property for isotropic turbulence is uniquely determined by λ_∞, since $\lambda_0 = 0$. Therefore, this theory determines the scaling laws of turbulence in terms of the characteristics of the most intermittent structure. Assuming that they are 1-D filamentary structures which lie in a boundary between two large eddies (of size ℓ_0) with a thickness of the order of the Kolmogorov length scale η, one obtains $\lambda_\infty = -2/3$, and predicts (She & Leveque 1994) the whole set of the scaling exponents ζ_p for the pth order velocity structure function:

$$\langle \delta u_\ell^p \rangle \sim \ell^{\zeta_p}, \qquad \zeta_p = \frac{1}{9}p + 2\left(1 - (\frac{2}{3})^{p/3}\right). \tag{4.2}$$

The formula (4.2) contains no adjustable parameter.

4.3 Comparison

During the last three years, both experimental and numerical studies have been conducted to test the model. Ruiz Chavarria *et al.* (1995a-b) have made the measurements in laboratory flows and have calculated quantities to test specifically the assumption (4.1), the assumption about the symmetry or the hierarchy. They claims that "... the hierarchy of the energy dissipation moment, recently proposed by She & Leveque for fully developed turbulence is in agreement with experimental data ...". Their method of calculation even leads to a direct determination of the parameter β which is in agreement with the proposed one ($\beta = 2/3$) for the isotropic turbulence. Furthermore, the measurements of the scaling exponents ζ_p in various flows carried out in several laboratories, e.g., in turbulent wakes (Benzi *et al.* 1994, 1995, 1996a-b), in grid turbulence (Herweijer & van de Water 1994), and in wind tunnel turbulence (Anselmet *et al.* 1984), and in jet turbulence (Noullez *et al.* 1997) are all consistent with (4.2) with remarkable accuracy. Finally, direct numerical simulations of the isotropic Navier-Stokes turbulence also accurately support

(4.2) (Cao *et al.* 1996). Some of these comparisons are reported below in Table 1.

There have been skepticisms about the meaning of the comparison. In this regard, we like to make the following remarks. First, the scaling exponents of the longitudinal velocity structure functions in a far-field of fully developed turbulent open flow have been measured in several flow environments, and the results are generally consistent (see Table 1). In other words, these experimental values are robust and stable. Recently, Belin, Tabeling & Willaime (1996) have reported the measurement of the scaling exponents in a closed flow system, which show values somewhat below the above reported ones $\zeta^{(4)}$. The measurements in the Taylor-Couette flow (another closed flow system) by Swinney's group at Texas also seem to show the same trend. The reason for this discrepancy is not yet clear. One possibility is that there is a systematic deviation of the scaling exponents between open and closed system, due to the interaction with the wall-ejected structures and strong rotation (both system develop strong swirls).

Order p	$\zeta_p^{(1)}$	$\zeta_p^{(2)}$	$\zeta_p^{(3)}$	$\zeta_p^{(4)}$	$\zeta_p^{(5)}$	SL Model ζ_p
1	–	0.37	–	–	0.362 ± 0.003	0.364
2	0.71	0.70	0.70 ± 0.01	0.70	0.695 ± 0.003	0.696
4	1.33	1.28	1.28 ± 0.03	1.26	1.279 ± 0.004	1.279
5	1.65	1.54	1.50 ± 0.05	1.50	1.536 ± 0.01	1.538
6	1.8	1.78	1.75 ± 0.1	1.71	1.772 ± 0.015	1.778
7	2.12	2.00	2.0 ± 0.2	1.90	1.989 ± 0.021	2.001
8	2.22	2.23	2.2 ± 0.3	2.08	2.188 ± 0.027	2.211
9	–	–	–	2.19	–	2.407
10	–	2.59	–	2.30	–	2.593

Table 1. Scaling exponents ζ_p of the *pth* order velocity structure functions measured in a wind tunnel turbulence$^{(1)}$ (Anselmet *et al.*, 1984), in a wake turbulence$^{(2)}$ (Benzi *et al.*, 1994), in a jet turbulence$^{(3)}$ (transverse velocity structure function) (Noullez *et al.* , 1996), in a low temperature helium experiment$^{(4)}$ (Belin *et al.* 1996), and in an isotropic Navier-Stokes turbulence simulation$^{(5)}$ (Cao, Chen & She, 1996). The SL model reads $\zeta_p = p/9 + 2(1 - (2/3)^{p/3})$.

Secondly, the Extended Self-Similarity property in turbulence (Benzi *et* *al.* 1993; Stolovitzky & Sreenivason 1993; Benzi *et al.* 1994; Briscolini *et al.* 1994) has greatly enhanced the accuracy of the measurement. Although the mechanism is not yet clear, the fact that it is a useful property in measuring scalings which leads no detectable distortion of the measured value is widely accepted. So the reported values in Table 1 are quite reliable. The good agreement can hardly attributed to pure coincidence.

Thirdly, it is fair to regard the comparison as a consistency check which is clearly positive. The fact that there exists other cascade ansatz which produce, with multiple adjustable parameters, a fit of the same quality does not invalid the present description. The advantage of the present model is its simplicity, the physical connection to flow structures, and its predictability of non-universal features of scalings as we will discuss later. It is also applicable to a variety of other turbulent systems with cascade dynamics. In short, it is worthy of further study.

4.4 Application to the GOY Shell-Model

While recognizing that a deductive theory of turbulence from the Navier-Stokes turbulence is highly desired, it is also important to examine carefully other systems exhibiting some essential features of the Navier-Stokes turbulent dynamics. These features include, from the present phenomenological understanding, the existence of an inertial range of scales of cascade (driven by the inertial force or the nonlinear convective term). The study of the other systems will allow one to identify the essential ingredients in the NS system such as the conservation laws, etc., which governs the cascade dynamics. More importantly, it will stimulate the development of a general theoretical framework for nonequilibrium systems presenting critical and scale invariance properties. This has been the essential motivation behind the study of a dynamical-system model of turbulence, namely the GOY shell model (see also, Kadanoff *et al.* 1995).

The GOY shell-model is a finite-dimensional dynamical system, which was introduced by Gledzer (1973) with an important extension made by Yamada and Ohkitani (1987, 1989) later introducing phase dynamics with complex variables. The dynamics are governed by the following set of ordinary differential equations:

$$\left(\frac{d}{dt} + \nu k_n^2\right)u_n = f_n + (a_n u_{n+1}^* u_{n+2}^* + b_n u_{n+1}^* u_{n-1}^* + c_n u_{n-1}^* u_{n-2}^*). \quad (4.3)$$

Here, $\{u_n\}_{0,1,...N-1}$ is a set of complex variables which model the Fourier space excitations in shells of wavenumbers $k_n = k_0\lambda^n \leq k < k_{n+1}$, f_n is a driving force usually acting on some small wavenumber shells, e.g. $f_n = f_2\delta_{n,2} + f_3\delta_{n,3}$. The term $\nu k_n^2 u_n$ models the viscosity damping with the kinematic viscosity ν.

At very small ν, the dynamics are essentially inviscid at small wavenumber shells where the nonlinear coupling (r.h.s. of (4.3)) make a chain linking the fluctuations at different wavenumber shells. At those shells (small n), the moments of the velocity fluctuations, $\langle |u_n|^p \rangle$, vary with the wavenumber in power law, $k_n^{-\zeta_p} \sim \ell_n^{\zeta_p}$. A number of studies have shown that the GOY shell model show very similar scaling laws as the 3-D Navier-Stokes equations (see e.g. Kadanoff *et al.* 1995 for more references). Leveque & She (1997) have made a careful study of the scaling exponents in the GOY shell model by a set of long numerical integrations of (4.3). The large sample size of the statistical fluctuation data has enable a detailed study of the convergence of the moments and the exponents. The scaling exponents ζ_p so measured are compared to the predictions of the Hierarchical Structure model as well as other cascade models (log-Normal, log-Stable, p-model, etc.). The conclusion is evident that the functional dependence of ζ_p are better represented by the Hierarchical Structure model (Leveque & She 1997). In Table 2, we report the comparison of ζ_p.

4.5 Other Predictions and Confirmation

The interesting feature of (4.2) is that the parameters determining the set of exponents ζ_p depends only on the properties of the most intermittent structures. These most intermittent structures at the length scale ℓ are theoretically defined, for e.g. the coarse-grained energy dissipation, by the limit $\lim_{p\to\infty} \epsilon_\ell^{(p)}$. In practice, this limit depends on the sample size of the data which is collected to describe the (spatio-temporal) ensemble of turbulence[1]. In the GOY shell model, we have collected enough sample so that for a certain range of scales, the limit has converged. In many other cases, $\epsilon_\ell^{(\infty)}$ depends on the sample size. However, its scaling exponent λ_∞ ($\epsilon_\ell^{(\infty)} \sim \ell^{\lambda_\infty}$) may depend more weakly on the sample size. Even when this dependence exists, it may be important to discover how it controlls the dependence of the measured scaling exponents for high order moments, e.g., ζ_p for large p.

[1] When the time-ensemble is considered, the limit $\lim_{p\to\infty} \epsilon_\ell^{(p)}$ is amount to calculate

$$\lim_{p\to\infty} \frac{\int_0^T \epsilon_\ell^{p+1}(t)dt}{\int_0^T \epsilon_\ell^{p}(t)dt}$$

which is well-defined for any finite large T. The sample size dependence will be reflected by the T-dependence.

Order p	ζ_p/ζ_3 (GOY)	$0.125p + 1.49(1 - 0.58^{p/3})$ (SL)
1	0.375 ± 0.005	0.372
2	0.705 ± 0.003	0.703
3	1.000	1.000
4	1.268 ± 0.006	1.268
5	1.512 ± 0.014	1.513
6	1.738 ± 0.026	1.737
7	1.946 ± 0.040	1.946
8	2.141 ± 0.058	2.140
9	2.323 ± 0.078	2.323
10	2.50 ± 0.10	2.50
11	2.66 ± 0.13	2.66
12	2.82 ± 0.15	2.82
13	2.97 ± 0.18	2.97
14	3.12 ± 0.21	3.12
15	3.26 ± 0.25	3.27
16	3.40 ± 0.28	3.41
17	3.54 ± 0.32	3.55
18	3.67 ± 0.36	3.68
19	3.80 ± 0.40	3.82
20	3.94 ± 0.44	3.95

Table 2. Scaling exponents ζ_p of the *pth* order velocity structure functions measured in the GOY shell model (Leveque & She, *Phys. Rev. E*, in press, 1997) compared with the Hierarchical Structure model of She and Leveque (SL). The parameters in the Hierarchical Structure model for the GOY shell-model is slightly different from those suggested for the Navier-Stokes turbulence. Those parameters are directly measurable from the data set.

It is difficult to draw a specific line between the reliable ζ_p's and those ζ_p's which have not yet fully converged. Instead of making arbitrary determination, we suggest to use the approximate information of the most intermittent structures (λ_p or γ) to characterize the whole set of measured scaling exponents. The basis for this proposal lies in the fact that the symmetry linking the most intermittent structures and less intermittent ones are more fundamental to turbulence, as we explain now.

The general formula for ζ_p reads

$$\zeta_p = \gamma p + C_0(1 - \beta^p), \tag{4.4}$$

where γ is similar to the smallest Hölder exponent of the velocity field when the scale varies in the inertial range, and C_0 is the co-dimension of the set of spatial points for which this exponent is realized.

The two parameters γ and C_0 are measurable in any finite sample of data[2], and then characterize the most intermittent structures detected within a finite spatial-temporal domain (where the ensemble is defined). The interesting fact is that there exist quantities which do not depend on γ and C_0. For instance, it is easy to verify that

$$\rho(p, q; p', q') = \frac{\zeta_p - p/p'\zeta_{p'}}{\zeta_q - q/q'\zeta_{q'}} = \frac{q'}{p'}\frac{p'(1-\beta^p) - p(1-\beta^{p'})}{q'(1-\beta^q) - q(1-\beta^{q'})}. \tag{4.5}$$

These quantities $\rho(p, q; p', q')$ described the relative scaling exponents between the normalized moments (p, p') and (q, q'). The fact that $\rho(p, q; p', q')$ depend only on β which characterizes the symmetry in the hierarchy suggests a method to verify the correctness of the hierarchy without a massive data set. If the symmetry is indeed more fundamental, then we may also observe a strong universality property of $\rho(p, q; p', q')$ compared to ζ_p.

Ruiz Chavarria *et al.* (1995a-b) have carried out experimental test specifically on the symmetry property of the model, and their results have fully confirmed its correctness. Benzi *et al.* (1996) have reported the Generalized Extended Self-Similarity (GESS) property from experimental results that $\rho(p, q; 3, 3)$ have a remarkable universal behavior for turbulent flows near a boundary layer, with and without a shear, etc. Their results are fully consistent with the prediction of the Hierarchical Structure Model. We believe that the model gives a plausible physical explanation of the ESS and GESS property.

An important practical interest of this universality result (GESS) is to allow us to differentiate among various turbulent systems. In practical situations, the properties (γ and C_0) of the most intermittent structures vary with the sample size, and also with space location and direction when the flows are not homogeneous and isotropic. According to the Hierarchical Structure Model, we can still have some reliable scaling laws among normalized moments reflecting the intrinsic symmetry. These scaling laws will allow us to

[2] We will discuss the measurability of C_0 later

determine γ or C_0 as a function of space location or direction, and hence get relevant physical information about the flow from the measurement of the scaling exponents. Deriving the structure information from the statistical measures is the unique advantage of the Hierarchical Structure Model.

4.6 Fluid Structures and Scalings: More Comments

It need to be emphasized that the most intermittent structures discussed above are, theoretically speaking, not directly the ones which are visualized in flow experiments and in numerical simulations. Instead, they are defined as the serie of structures at the inertial-range scales as follows. First, obtain the coarse-grained dissipation field ϵ_ℓ at scale ℓ. Second, identify $\epsilon_\ell^{(\infty)}$ as the spatial locations which contribute the most to the ratio of the moments $\lim_{p\to\infty}\langle\epsilon_\ell^{p+1}\rangle/\langle\epsilon_\ell^p\rangle$. These spatial points form a set whose volume depend on the scale ℓ. The concept of the co-dimension $d-D$ is an abstraction of the fact the volume changes in ℓ as ℓ^{d-D} where d is the dimension of the space and D is dimension of the set.

As the coarse grained scale decreases $\ell \to \eta$ to be close to or below the dissipation cutoff scale η, we can also identify the set $\epsilon_\eta^{(\infty)}$ as the most intermittent structure at scale η. These structures may be close to the iso-surfaces at a high threshold of the original, non-coarse-grained field ϵ. Furthermore, their geometry may not change significantly as we further coarse-graining the field from scale η to $\ell > \eta$. If these two conditions are satisfied, then the geometry of the visualized structures give a qualitatively correct estimate of the co-dimension of the most intermittent structures. We can hope that when the turbulence velocity field change smoothly as ℓ going through η (e.g. no appreciable oscillations around high peaks of the velocity fluctuations), then these conditions are satisfied.

However, in some arbitrary mathematical problems such as the solution of the Navier-Stokes equation with hyperviscosity, strong oscillations in the physical field may appear at very small scales (around the dissipation cutoff scale η) due to the non-positiveness of the hyperviscous "diffusion". The consequence is that the smooth transition from the inertial-range scale ℓ to η is interrupted. In this case, the relation between the characteristics of the inertial-range most intermittent structures with the visualization of the dissipation range quantity such as ϵ become elusive.

5 Further Theoretical Development

The Hierarchical Structure model has also been examined from a more theoretical standpoint. The main results are that the symmetry (4.1) can be exactly realized via a simple random cascade process called log-Poisson (Dubrulle

1994, She & Waymire 1995), and it follows also from an invariance property in the transformation of the frame of reference in a new coordinate systems, the amplitude-scale system (Dubrulle and Graner 1996a-b, She and Leveque 1997).

5.1 Log-Poisson Cascade

It is shown by Dubrulle (1994) and independently by She & Waymire (1995) that (4.1) can be exactly realized by a random multiplicative cascade process, called Log-Poisson. Let the integral-scale eddies be represented by the coarse-grained energy dissipation ϵ_{ℓ_0} (a random variable). Let the small-scale eddies at any given length scale ℓ be generated by

$$\epsilon_\ell = (\frac{\ell}{\ell_0})^\gamma \beta^n \epsilon_{\ell_0} \tag{5.1}$$

where n is an independent Poisson random variable with a mean λ:

$$P(n) = e^{-\lambda}\frac{\lambda^n}{n}, \qquad n = 0, 1, 2, ... \tag{5.2}$$

It can be deduced from (5.1) that

$$\langle \epsilon_\ell^p \rangle = (\frac{\ell}{\ell_0})^{\gamma p} \sum_n \beta^{np} P(n) \langle \epsilon_{\ell_0}^p \rangle. \tag{5.3}$$

Then, (4.1) follows.

According to (5.1), a large-scale eddy has a number of possibilities when it is transformed to a smaller one. The largest amplitude is achieved at $n = 0$ because $\beta < 1$; other smaller amplitude events are obtained by multiplying an integer number of β factors. The $n = 0$ event is of special interest: it varies with scale as ℓ^γ and the probability of finding it is $e^\lambda \sim \ell^{C_0}$. This is the most intermittent event. When $C_0 > 0$, or the strongly excited events reside in a smaller (fractal) set, $e^\lambda \to 0$ as $\ell \to 0$. This is the intermittency, or anomalous scaling, because the whole space is occupied by less excited events. When the multiplication of β factors acts as a Poisson point process, the symmetry (4.1) is exactly realized.

5.2 Log-Poisson Cascade and Other Cascade Models

Compared to other discrete cascade models proposed earlier (e.g. Meneveau & Sreenivason 1987), the log-Poisson has the following features: First, the cascade from ℓ_0 to ℓ_1 and then to ℓ_2 is identical to the cascade from ℓ_0 to ℓ_2. This can be shown as the follows. Let $\mathcal{W}_{01} = (\ell_1/\ell_0)^\gamma \beta^{n_1}$ and $\mathcal{W}_{12} = (\ell_2/\ell_1)^\gamma \beta^{n_2}$. Then, it can be shown, by working with $\log \mathcal{W}$, that $\mathcal{W}_{02} =$

$\mathcal{W}_{01}\mathcal{W}_{12} = (\ell_2/\ell_0)^\gamma \beta^N$ where N is again a Poisson random variable, and moreover $\langle N \rangle = \langle n_1 \rangle + \langle n_2 \rangle$. This proof is valid for any arbitrary ℓ_1 and ℓ_2, which removes one important arbitrariness in defining the step of cascade ℓ_1/ℓ_0 or ℓ_2/ℓ_1. This arbitrariness is a major shortcoming of the previous discrete cascade models when used for describing such a continuous scaling process as turbulence. In this regard, the log-Poisson process is self-consistent and the reason for its success lies in its log-infinite divisibility property (see She & Waymire 1995).

Secondly, the Log-Poisson cascade picture does not have any difficulty which other log-infinitely divisible process such as the Log-Normal model (Kolmogorov 1962) or the Log-stable model (Kida 1989) has, namely, the physically unacceptable behavior of ζ_p for large p. It is true that both the Log-Normal model and Log-stable model, with a certain choice of the parameters (which by the way cannot be estimated based on physical properties of the flow), agree with some experimental values for a moderate range of p. We believe that this is the evidence of a good approximation of the model over a range of p, just as the approximation of a smooth function locally by a quadratic form. However, both models make very strong predictions about the asymptotic behavior of ζ_p at large p, or equivalently saying, the behavior of the fluctuation events at very large amplitude. This behavior requires that the velocity be unbounded in the limit of vanishing viscosity which create a mathematical inconsistency as to work with the incompressible Navier-Stokes equation (Frisch 1991). Strictly speaking, there is no evidence supporting the divergence of the velocity in the limit of vanishing viscosity for the three-dimensional Navier-Stokes equation in a periodic domain under a deterministic forcing at low wavenumbers. And it is virtually impossible for experiments to provide a convincing test of the assertion either.

On the other hand, the measured anomalous, or non-Kolmogorovian scalings at moderately large p, in both the Navier-Stokes flows and laboratory flows, have now quite solid evidence. It seems pointless to base the theory of the anomalous scalings on a model whose strong prediction can "never" be checked, without mentioning its unlikelihood from purely a stochastic process point of view (Mandelbrot 1974). By contrast, the log-Poisson model makes no fixed assertion about the large p behavior. It says that the large p behavior is the property of the most intermittent structures currently in the spatio-temporal domain in question. This behaviors could vary case by case, giving rise to some appearant scattering of the measured scaling exponents. On the other hand, there is a stable symmetry which is built by the log-Poisson cascade between the scaling (if it is there) of the most intermittent structures and of other less excited ones. This symmetry is more intrinsic and visible, and is therefore experimentally detectable already at moderately large p. This description enjoys the simplicity and rely on no speculative basis in "unreachable" asymptotic.

5.3 Application to the Navier-Stokes Turbulence

The theory of the log-Poisson cascade has an interesting application, that is, find the probability density function (PDF) of small scale fluctuations from the PDF of large scale ones. Note first that any small-scale fluctuations ϵ_ℓ is equal *in law* to the large-scale fluctuations ϵ_{ℓ_0} multiplied by an independent random variable $\mathcal{W}$. Therefore,

$$\log \epsilon_\ell \stackrel{\ell}{=} \log \epsilon_{\ell_0} + \log \mathcal{W}. \tag{5.4}$$

Consequently,

$$P(\log \epsilon_\ell) = P(\log \epsilon_{\ell_0}) \otimes P(\log \mathcal{W}), \tag{5.5}$$

where $\otimes$ denotes a convolution.

One particular application is that at large Reynolds numbers and at very small scales ($\ell \to 0$), ϵ_ℓ being approaching to ϵ is fluctuating much more intermittently than ϵ_{ℓ_0} (the averaged dissipation over a large domain). In this case, $P(\log \epsilon_{\ell_0})$ behaves approximately as a δ-function, compared to $P(\log \mathcal{W})$. In this case, $(\log \epsilon_\ell)$ will mimic closely a log-Poisson form, a smooth version of the discrete (atomic) distribution function $P(\log \mathcal{W})$.

Leveque and She (1996) have tested this prediction. A forced 3-D incompressible Navier-Stokes equations under periodic boundary conditions are integrated numerically using a pseudo-spectral method. With a resolution of 256^3, a sample of stationary isotropic turbulence is generated with a Taylor microscale Reynolds number $R_\lambda \approx 120$. The probability density function $P(\log \epsilon)$ are then measured and compared to a log-Poisson fit. The result is quite satisfactory, as presented in Fig. 1.

Recently, Novikov (1994) introduced a gap argument against (5.1). In fact, there is a big difference between the breakdown coefficient which he used and the experimentalists measured and the random multiplicative coefficients $(\frac{\ell}{\ell_0})^\gamma \beta^n$ used here, because the former can not be truly random and independent of the large eddies (ϵ_{ℓ_0}). Even a slight dependence will largely affect the theoretical conclusion, specially for large p moments. A more careful analysis taking into account the slight statistical dependence between the breakdown coefficient and the large eddies, the Novikov's equality becomes an inequality which resolves the controversy raised by him. Of course, we can not claim the correctness of the formula (4.4) at this stage, but it seems unlikely that the gap argument presents a serious threat.

5.4 Invariance Principle

Recently, She & Leveque (1996) proposed another derivation of the hierarchical symmetry based on an invariance principle using similar reasoning as in the theory of special relativity. The work was stimulated by an earlier work of Dubrulle and Graner (1996a-b). Instead of addressing the transformation

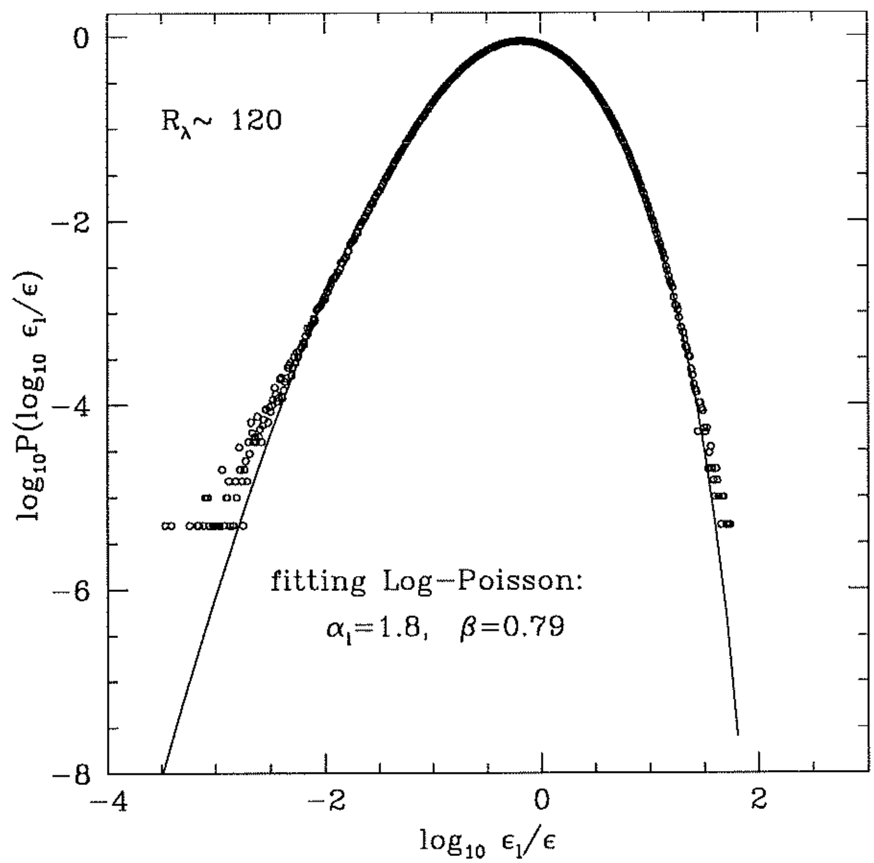

Fig. 1. The probability density function of the energy dissipation measured in a simulation of the Navier-Stokes isotropic turbulence, compared with the PDF obtained by a fit with the log-Poisson model.

of the PDFs from the large scale to small scale as in the Log-Poisson case, we propose to address the transformation of moments. Let ϵ_ℓ denote the coarse-grained energy dissipation where ℓ continuously changes from ℓ_0 to η. At each scale ℓ, the fluctuation of ϵ_ℓ has a maximum amplitude, called $\epsilon_\ell^{(\infty)}$. We make the following fundamental assumptions:

H1 : There is no characteristic length scale other than the integral scale ℓ_0.

H2 : There is no characteristic fluctuation amplitude other than $\epsilon_\ell^{(\infty)}$.

The assumption (H1) is the similarity property in scale, and (H2) is the similarity property in amplitude. Since (H1) and (H2) are so much similar, it is natural to set up a coordinate system in scale and amplitude so that the two similarity properties can be fully explored. Following Dubrulle & Graner (1996a), the new coordinate system is to make the following identification:

$$X(\cdot) = \log E(\cdot); \quad T = \log \ell. \tag{5.6}$$

Here $E(\cdot)$ refers to as the expectation value of a random variable. In this system, the amplitude-scale becomes "space-time". An arbitrary stochastic field will result in a path (or trajectory) in this coordinate system. In particular, a scaling field such as the increment of a fractional Brownian motion of exponent h such that $E(|\Delta x|) = E(|x(t+\ell) - x(t)|) \sim \ell^h$ will be described as a straight line of constant speed h.

The assumption (H1) implies generally that all statistical average quantities behave as a power-law in ℓ/ℓ_0 (at least in leading order). The assumption (H2) suggests that if we introduce $\Pi_\ell = \epsilon_\ell/\epsilon_\ell^{(\infty)}$, then there exists no characteristic amplitude between 0 and 1 at each ℓ. It then follows that the field Π_ℓ^p is a scaling field, that is, $E(\Pi_\ell^p) = \ell^{V_p}$. Next, we conjecture:

H3 : The field Π_ℓ^p (for any p) defines a so-called inertial frame of reference with an intrinsic speed V_p. In other words, V_p is independent of whether the observer is in a rest or moving frame of reference.

A specific proposal of the meaning of the moving frame of reference is given in She & Leveque (1996). In the moving frame of speed V_p (defined by the scaling field Π_ℓ^p), the space coordinate of another scaling field Π_ℓ^q is proposed to be measured in the following way:

$$X_q^{(p)} = \log\left(E(\Pi_\ell^p \Pi_\ell^q)/E(\Pi_\ell^p)\right). \tag{5.7}$$

In other words, (5.7) suggests that the "space" coordinate measurement in a moving frame corresponds to an average with respect to a weighted probability density function by a factor Π_ℓ^p. As p increases, more weight is put on the high amplitude fluctuations. With this restriction, it can be shown (She & Leveque 1996) that the "space" and "time" coordinates will satisfy a transformation law when the observer moves from one frame of reference to another:

$$X' = X - WT; \quad T' = \left(1 - \frac{W}{V_\infty}\right)T, \tag{5.8}$$

where W is the speed of the moving frame. From (5.8), we can derive a composition law for the relative speeds:

$$V_{p+q}^{(0)} = V_p^{(0)} + V_q^{(p)} - \frac{V_p^{(0)} V_q^{(p)}}{V_\infty}. \tag{5.9}$$

However, (H2) suggests the equivalence of all inertial frames of reference, since they simply correspond to the emphasis on different amplitudes and there is no prefered amplitude. This justifies the second part of (H3) which can be more explicitly stated as $V_q^{(p)} = V_q^{(p')}$ for any p and p'. Substituting $V_q^{(p)} = V_q^{(0)}$ into (5.9) leads to the solution for V_p:

$$V_p = V_\infty(1 - \beta^p), \tag{5.10}$$

where $V_\infty = C_0$ plays the role of the "speed of light" as in the theory of special relativity. Eq. (5.10) is identical to (4.4) in its nonlinear part.

It is interesting to see that the Hierarchical symmetry (4.1) follows from such an elegant argument of the invariance of "speed" $V_q^{(p)} = V_q^{(p')}$ which expresses the fact that all frame of references are equivalent.

5.5 Analogy with the Theory of Special Relativity

Technical assemblance between the discussion in the last section and the theory of special relativity is striking. We can make a few more comments about what we learn from this analogy.

The theory of special relativity challenged the classical picture of Newton which involved an absolute and homogeneous space, within which things changed in an absolute and homogeneous time. In particular, the constancy of the speed of light measured in all moving frames of reference imposes a special connection between the space and time coordinate of things under study. The Lorenz transform merely expressed this "relativity" in concrete mathematical form and gave rise to many predictions testable in physical environment.

The analogy between our present treatment and the theory of special relativity does not imply any change in the understanding of the Newtonian space-time structure for the fluid mechanics. The analogy is seen in the new "space-time" coordinate system for the convenience of describing the statistical structure of turbulence, that is, how the statistical averages (moments) change as the length scale decreases. When the length scales ("time") and the moments ("space") are normalized or scaled properly (setting up the origin), the theory of the last section indicates that turbulent stochastic field (the kinematic of event) is such that the two variables of the length scale and the moment amplitude are related.

It will become more clear if we contrast it with the Kolmogorov 1941 (K41) description within the same framework. In K41, there is no upper characteristic fluctuation amplitude, so $V_p = \zeta_p = hp$ ($h = 1/3$). It is easy to check that $V_{p+q} = V_p + V_q$, $X_q^{(p)} = (V_{p+q} - V_p)T = X_{p+q} - V_pT$. Therefore, we have a system of Galilean transformation which corresponds to the prescription that Kolmogorov (1941) gave to turbulence field. Speaking in different words, Kolmogorov described an absolute and homogeneous "space" of amplitude of fluctuations in the sense that big amplitude fluctuations cascade to small scales in identical way as small amplitude fluctuations.

In contrast, we find important to recognize the role of the most singular structures for the evolution of the whole statistical ensemble of fluctuations from large to small scales (the kinematic in the new "space-time" coordinate system). This is closely parallel to recognizing the existence of the speed of light, which consequently make the new "space" and "time" united. The fact that we have a different law of transformation from the Lorenz law is due to the lack of symmetry between the positive speed and negative speed, one specifying divergent changes and other convergent changes as the scale decreases.

6 Universality and Non-Universality

The issue of the universality is one of the most important question for turbulence (see e.g. Nelkin 1994). When the medium becomes increasingly complicated, physical quantities show richer behavior, and theories describing the relationship between these quantities shows inevitably more complexity. Only the discovery of certain universality would lead to the hope for some simple and elegant theoretical description. This is the basic harmony that the complex universe display.

The universal behavior does not invoke detailed mechanisms specific to the dynamics, and should invite a theoretical description of general character. For turbulence, we believe that there are some general principles due to simply the non equilibrium and the strong nonlinearity. Because of the non equilibrium, the energy cascades from large to small scales. Because of the strong nonlinearity, the inertial range is formed. Furthermore, we conjecture that strong nonlinear interaction also leads to a strong phase mixing which is the origin of the symmetry (4.1), or of the new universal physical quantities such as (4.5). At present, the Hierarchical Structure Model is consistent with all existing observations. But it presents only the first step in grasping the universal aspect. More important work will be ahead to elucidate the "first" principle behind this universality.

Next, we argue that the understanding of the universality is the only way to address the non-universal behavior. We believe that ζ_p may not be universal. Experimentally observed values show certain scattering; we think that it is not a simple statistical convergence problem. There is more physical meaning behind. For example, we conjecture that it may reflects the fluctuation of the most intermittent structures captured by the given sample. In any sample set, they are the largest fluctuation events and also the rarest events. They change easily from sample to sample, from environment to environment. Because ζ_p, even for the moderately large p, is sensitive to them, we observe scattering. This explanation, if being correct, indicates that ζ_p

is not the most robust measure. The Hierarchical Structure Model suggests to study $\rho(p, q; p', q')$ instead, which describe theoretical the statistical link between the most intermittent structures and less intermittent structures.

The practical interest of studying $\rho(p.q; p', q')$ is to find a better parameterization for ζ_p which is physical. If indeed the property of the most intermittent structures determines ζ_p, then the systematic variation of ζ_p would reveal a change of physical environment, and thus the physical origin of the non-universality is revealed. We believe that a physical theory of the scaling should explain the physical origin of the non universal behavior of ζ_p (among others). In other words, the theory should contain parameters which can either be direct measurable or can be estimated by plausible theoretical arguments. The Hierarchical Structure Model has offered a plausible candidate.

Acknowledgment This work presented here is a continuation of the collaboration with Dr. E. Leveque and E. Waymire. I have also greatly benefited from discussions with Dr. R. Benzi and S. Chen. I am also grateful to the support by the Office of Naval Research which has enable a number of scientific interactions relevant to the development of the work.

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