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ABSTRACT

Total variation (TV) minimizing image restoration is a fairly new approach to image restoration, and has been shown both analytically and empirically to be quite effective. Our primary concern here is to develop a spatially *adaptive* TV minimizing restoration scheme. One way of accomplishing this is to locally weight the measure or computation of the total variation of the image. The weighting factor is chosen to be inversely proportional to the likelihood of the presence of an edge at each discrete location. This allows for less regularization where edges are present and more regularization where there are no edges, which results in a spatially varying balance between noise removal and detail preservation, leading to better overall image restoration. In this paper, the likelihood of edge presence is determined from a partially restored image. The results are best for images with piecewise constant image features.

Keywords: Total Variation, Adaptive, Image Restoration

1. INTRODUCTION

Total variation (TV) minimizing function regularization was recently introduced[1] for use in image restoration. The approach is to find an approximation, $u = u(\vec{x})$, to the true image, $u_{true} = u_{true}(\vec{x})$, given the degraded (e.g. noisy) image, $u_0 = u_0(\vec{x})$, where $u_0 = u_{true} + \eta$, where $\eta = \eta(\vec{x})$ represents the noise or other unwanted characteristics in the image. The basic idea of Rudin, Osher and Fatemi[1] is to minimize the total variation $TV(u)$ in the image, where

$$TV(u) \equiv \int |\nabla u(\vec{x})| \, d\vec{x}, \tag{1}$$

while preserving some fit to the original data u_0. The mathematical formulation of this problem which we consider in this paper is the noise-*constrained* problem

$$\min_u TV(u) \quad \text{subject to} \quad \|u - u_0\|^2 = \sigma^2, \tag{2}$$

where the noise level σ^2 is assumed to be known or fairly accurately estimated.

The main advantage that TV image restoration has over other image restoration techniques is that it penalizes neither discontinuities (i.e. edges) nor smoothness in the restored image u. TV regularization has been shown, both analytically[2,3] and numerically[1,4] to be quite effective in image restoration, especially for restoring images with piecewise constant features.[2] In particular, it preserves *exactly* the location of edges, which is often extremely important information. Still, because of the nature of the TV functional (1), TV restoration does not prefer discontinuous edges to smoothness. TV restoration looks for an approximation u to the original image u_0 which has a smaller total variation, but with no particular bias toward a discontinuous or smooth solution. In solving (2), it is the original data, u_0, as well as the estimated noise level, σ^2, which determine the sharpness or smoothness of the restored image. We note that TV image restoration can be viewed as a special model case[5] of the recently introduced class of

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anisotropic diffusion schemes,[6] to which a forthcoming issue of *IEEE Transactions on Image Processing* is devoted (scheduled for Spring, 1998).

At present, a primary concern in image restoration is to develop spatially *adaptive* restoration schemes. It is often the case that where there is more detail in an image, it is desirable to apply relatively *less* noise-removing restoration, in order to better retain the detail, and conversely where there is less detail in the image (e.g. within a piecewise constant region), it is desirable to apply relatively *more* restoration. In solving (2) there is a certain balance between noise removal and feature or detail preservation. This balance is determined primarily by the noise level σ^2, and is *constant* throughout the image. For TV minimizing image restoration, the key to spatially adaptive restoration is to allow this balance between noise removal and detail preservation to be spatially *varying*. Without this spatial adaptivity, smaller-scaled features can be compromised or even lost completely, as the effects of TV minimizing image restoration are inversely proportional to the scale of individual image features.[2]

At present, the authors are aware of a single proposal for an adaptive *TV minimizing* restoration scheme,[7] which to date has not been published. In this paper, we propose a spatially adaptive TV minimizing image restoration scheme where the adaptivity is realized by using a weighted TV functional

$$TV_\omega(u) \equiv \int \omega(\vec{x})|\nabla u(\vec{x})|\, d\vec{x}. \tag{3}$$

The corresponding minimization problem would then be

$$\min_u TV_\omega(u) \quad \text{subject to} \quad \|u-u_0\|^2 = \sigma^2. \tag{4}$$

A previous detailed analysis[2] of TV minimizing function regularization provides a theoretical justification for this approach.

We choose $\omega(\vec{x})$ based on the likelihood of the presence of an edge between any two neighboring discrete image locations. The weighting factor is chosen to be inversely proportional to the likelihood of the presence of an edge. This allows for less regularization where edges are present and more regularization where there are no edges, which results in better overall noise removal and detail preservation. The results are generally good, particularly for images with piecewise constant image features.

In the balance of the paper, we present our basic ideas in R^1 and give results for restoring noisy R^1 images, after which we extend our scheme to R^2 and give results for restoring noisy R^2 images.

2. ADAPTIVE RESTORATION IN R^1

We first discuss the weighted TV norm (3) in R^1. We next discuss how to choose the weighting factor, first by using *a priori* information and then automatically. Results of restoring noisy R^1 images are then given.

2.1. Weighted TV Norm in R^1

To accomplish adaptive TV miniminizing image restoration, we replace (1) with the weighted TV norm (3). In R^1, the discrete version of (1) is

$$TV(u) = \sum_{i=1}^{n-1} |u_x(i+\tfrac{1}{2})| = \sum_{i=1}^{n-1} |u_{i+1}-u_i|; \tag{5}$$

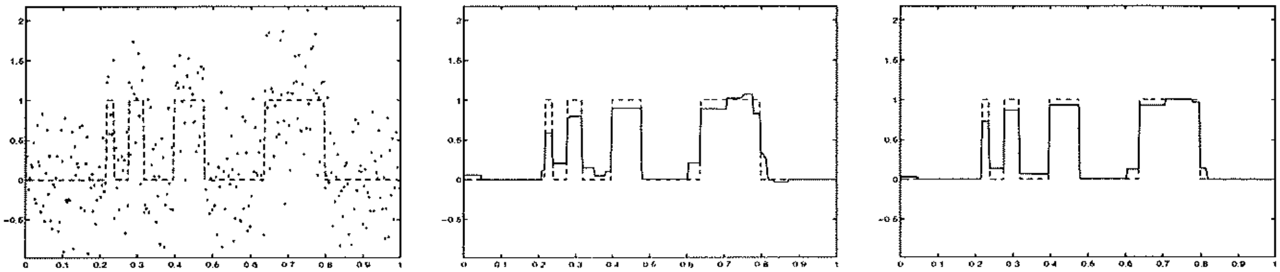
where $u_x(i+\frac{1}{2})$ represents $\frac{du(x)}{dx}|_{x=x_{i+\frac{1}{2}}}$. The weighted TV functional (3) is analogously

$$TV_\omega(u) = \sum_{i=1}^{n-1} \omega_{i+\frac{1}{2}} |u_{i+1}-u_i|. \tag{6}$$

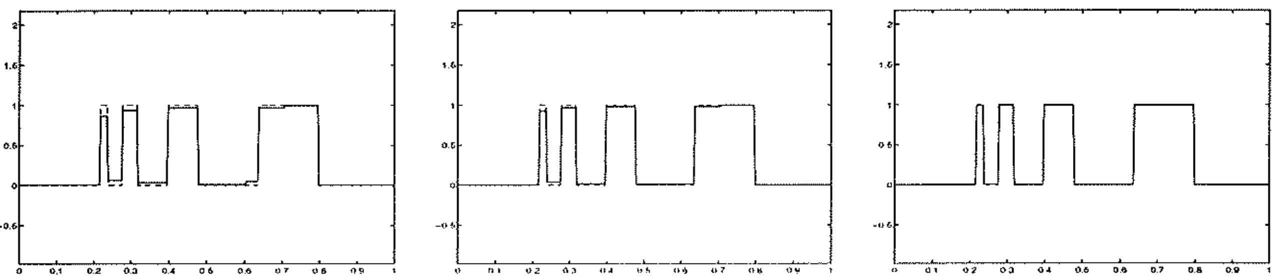
The idea then is to choose $\omega_{i+\frac{1}{2}}$ to be *smaller* where there is an edge between u_i and u_{i+1}, and conversely to choose $\omega_{i+\frac{1}{2}}$ to be *larger* where there is no edge. The motivation is to better preserve edges by allowing the variation in the image which is due to the edges, and to better remove noise where edges are not present by penalizing the variation in the image that is due to the noise. In regions of relatively moderate intensity change (i.e. smooth, non-piecewise constant image features), the choice of $\omega_{i+\frac{1}{2}}$ would be somewhere in between. This relationship can be written

$$\omega_{i+\frac{1}{2}} \sim \frac{1}{\text{Likelihood of an edge between positions } i \text{ and } i+1 \text{ in } u_{true}}.$$

2.2. Adaptive Restoration Based on Likelihood of Presence of Edges



(a) Noisy image. (b) Restored image, 1:1 ratio. (c) Adaptively restored, 2:1 ratio.



(d) Adaptively restored, 5:1 ratio. (e) Adaptively restored, 10:1 ratio. (f) Adaptively restored, 100:1 ratio.

Figure 1. A comparison of *non-adaptive* TV restoration (2) to *adaptive* restoration (4). We choose ω_{edge}, the value of ω where edges are present in the noise-free image, to be smaller than ω_{flat}, the value of ω where edges are not present. The ratio of ω_{flat} to ω_{edge} is 1:1 (i.e. non-adaptive), 2:1, 5:1, 10:1, and 100:1 for finding the restored image in (b), (c), (d), (e) and (f), respectively. The results are better for larger ratios of $\omega_{flat} : \omega_{edge}$. Table 1 gives the ISNR for each of the restored images.

In Figure 1 we give an example of adaptive restoration for a piecewise constant R^1 function, contaminated with Gaussian noise, SNR = 0 dB, where the edge locations are known *a priori*, to demonstrate the effectiveness of the our idea for choosing $\omega_{i+\frac{1}{2}}$. In finding each of the restored images shown in Figures 1(b) - (f), we have chosen

$$\omega_{i+\frac{1}{2}} = \begin{cases} \omega_{edge} & \text{if there } \textit{is} \text{ an edge (discontinuity) between positions } i \text{ and } i+1 \text{ in } u_{true} \\ \omega_{flat} & \text{if there } \textit{is not} \text{ an edge (discontinuity) between positions } i \text{ and } i+1 \text{ in } u_{true} \end{cases}$$

where $\omega_{edge} \leq \omega_{flat}$. We note that in solving the *constrained* problem (4), is is not the actual values of $\{\omega_{i+\frac{1}{2}}\}$ that are important, but rather their *relative* values, that is, relative to each other. We solve (4) using different ratios for $\omega_{flat} : \omega_{edge}$, where $TV_\omega(u)$ is as in (6). We find, as expected, that the adaptive restoration scheme is more effective for larger ratios of $\omega_{flat} : \omega_{edge}$. Because of the exact information which we had about edge location in this contrived example, we have

$$u \longrightarrow u_{true} \quad \text{as} \quad \frac{\omega_{flat}}{\omega_{edge}} \longrightarrow \infty, \tag{7}$$

Figure 1	$\omega_{flat} : \omega_{edge}$	ISNR
(a)	(Noisy Image)	0.00 dB
(b)	1:1	11.78 dB
(c)	2:1	16.09 dB
(d)	5:1	22.72 dB
(e)	10:1	28.21 dB
(f)	100:1	47.62 dB

Table 1. The improved SNR for each of the adaptively restored images (shown in Figure 1) found by solving (4), where the ratio of the weighting of (3) in the flat regions to the weighting at the edges is given by $\omega_{flat} : \omega_{edge}$. We note that the case in which $\omega_{flat} : \omega_{edge} = 1 : 1$ is simply the standard (non-adaptive) case.

if the mean of the noise is 0 in each of the piecewise constant regions (which we artificially enforced for this example). Figure 1 illustrates (7).

2.3. Automatically Defining $\{\omega_{i+\frac{1}{2}}\}$ in R^1

In defining $\{\omega_{i+\frac{1}{2}}\}$ in the preceeding example we knew *a priori* the location of the edges. In general we do not have to this information *a priori* (otherwise the problem is often already solved). Here our approach in determining the likelihood of an edge at any given location is based on examining the size of the jump between neighboring pixels of a partially or fully (non-adaptively) restored image. The reason for this is that a partially restored image can give us valuable information which can be used to determine the weighting factor ω, which we can then use to restore the noisy image with a spatially varying balance between noise removal and fit to the original data. Strong and Chan[2] found that the effects of TV minimizing image restoration are inversely proportional to scale, so that smaller-scaled features are sometimes compromised or completely lost in our attempt to remove noise from the image. Thus partial noise removal (i.e. a partially restored image) can potentially give us better information about the more detailed or smaller-scaled features than could a completely restored image, in which the levels of both noise and detail are decreased. The algorithm for implementing this approach is given in Table 2.

Algorithm for Adaptive TV Restoration in R^1

1. Find $\tilde{u}$ by solving
   $$\min_{\tilde{u}} TV(\tilde{u}) \quad \text{subject to} \quad \|\tilde{u} - u_0\|^2 = \tilde{\sigma}^2,$$
   where $0 < \tilde{\sigma}^2 \leq \sigma^2$.

2. For $1 \leq i \leq n-1$, define
   $$\omega_{i+\frac{1}{2}} = \frac{1}{|\tilde{u}_{i+1} - \tilde{u}_i| + \epsilon},$$
   where $\epsilon > 0$.

3. Find the adaptively restored image u by solving
   $$\min_{u} TV_\omega(u) \quad \text{subject to} \quad \|u - u_0\|^2 = \sigma^2.$$

Table 2. Adaptive TV minimizing image restoration scheme in R^1.

The first step is to solve the standard minimization problem (2) using a (possibly) smaller estimated noise level $\tilde{\sigma}^2 \leq \sigma^2$. Thus by solving (2) using $\tilde{\sigma}^2$ we expect to remove much of the noise while preserving the image features, particularly if the image is detailed. This information can then be used to define $\{\omega_{i+\frac{1}{2}}\}$ in Step 2. In Step 2, the parameter ϵ is chosen both to stabilize the numerical problem and to give us more control over how adaptive the scheme will be. The larger ϵ is, the less variation there is in the weighting factor $\omega(\vec{x})$ in (3). For ϵ very large, the adaptive scheme essentially becomes the standard scheme. On the other hand, if ϵ is chosen to be extremely small, the weighting factor $\omega(\vec{x})$ may be too adaptive, resulting in an image where discontinuous edges are artificially introduced, due to the corruption of the image from the noise. The choice of ϵ will be directly related to the range of the grayscale values in the image. Appropriate choices of ϵ will be briefly addressed in our subsequent analysis of our results in R^2. With $\{\omega_{i+\frac{1}{2}}\}$ defined, in Step 3 we again solve the constrained minimization problem, this time using the weighted TV functional.

We briefly comment about the extra computational cost of the adaptive scheme. In the standard TV minimizing restoration scheme, we solve the constrained minimization problem a single time. In the adaptive scheme we solve a constrained minimization problem twice. Thus the adaptive scheme could be about twice as expensive to apply as the standard scheme. However, we note that the partially restored image $\tilde{u}$ found in Step 1 can be used as a good initial guess for the iterative scheme used to find u in Step 3.* Because of this good initial guess for solving the minimization problem in Step 3, the computational work of the adaptive scheme as compared to the standard scheme is actually increased by *less* than a factor of 2.

2.4. Results in R^1

To illustrate the effects of this scheme, we again consider the noisy image from the previous example. We use the true and noisy images from the previous example, as shown in Figure 1(a), and we can compare the results of our scheme to the results of standard (i.e. non-adaptive) TV restoration, found in the previous example and shown in Figure 1(b). In Figure 2 are the adaptively restored images found by using the values $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.50, 0.75, 0.90, 1.00$, and $\epsilon = 1.00, 0.10, 0.01$. For this example the results are better for larger values of $\frac{\tilde{\sigma}^2}{\sigma^2}$ and smaller values of ϵ.

$\frac{\tilde{\sigma}^2}{\sigma^2} \setminus \epsilon$	1.00	0.10	0. 01
0.50	13.63 dB	15.58 dB	15.93 dB
0.75	13.55 dB	15.96 dB	16.45 dB
0.90	13.66 dB	16.86 dB	18.20 dB
1.00	13.56 dB	16.72 dB	18.23 dB

Table 3. The improved SNR for the adaptively restored images, using various ratios of $\frac{\tilde{\sigma}^2}{\sigma^2}$ and various values of ϵ. The ISNR for the standard (non-adaptive) restored image is 11.78 dB, as shown in Table 1, row (b). The restored images, shown in Figure 2, were found using the scheme given in Table 2.

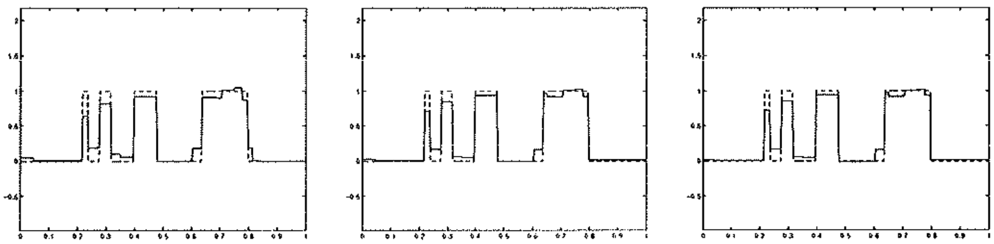
3. ADAPTIVE RESTORATION IN R^2

3.1. Weighted TV Norm in R^2

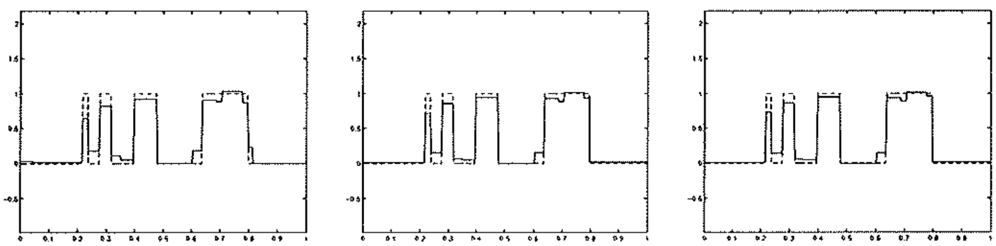
We now extend our discussion to images in R^2. For a discrete R^2 image $\{u_{i,j}\}$, $1 \leq i, j \leq n$, the most natural way of discretizing (1) is

$$TV(u) = \tfrac{1}{2}\{ \sum_{i=1}^{n-1}\sum_{j=1}^{n} \sqrt{[u_x(i+\tfrac{1}{2},j)]^2 + [u_y(i+\tfrac{1}{2},j)]^2} + \sum_{i=1}^{n}\sum_{j=1}^{n-1} \sqrt{[u_x(i,j+\tfrac{1}{2})]^2 + [u_y(i,j+\tfrac{1}{2})]^2} \ \}$$

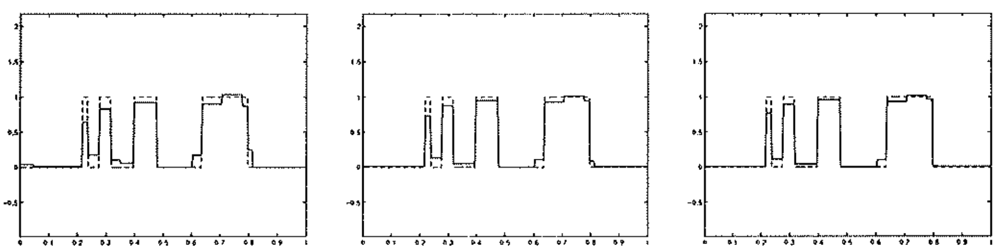
*The numerical problem that arises from solving the TV minimizing restoration problem (2) or (4) is nonlinear, and hence must be solved with an iterative solver, for which an initial estimate or guess is needed.



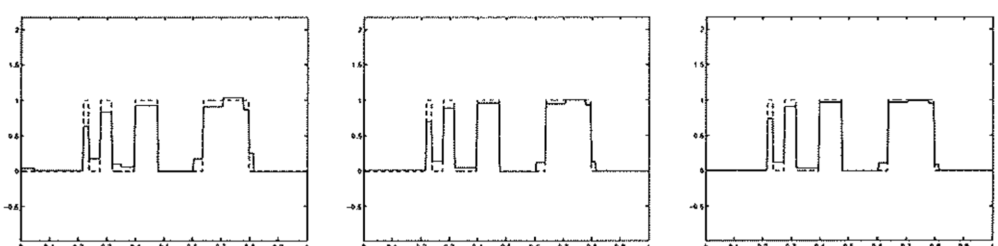
(a) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.50$, $\epsilon = 1.00$. (b) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.50$, $\epsilon = 0.10$. (c) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.50$, $\epsilon = 0.01$.



(d) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.75$, $\epsilon = 1.00$. (e) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.75$, $\epsilon = 0.10$. (f) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.75$, $\epsilon = 0.01$.



(g) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.90$, $\epsilon = 1.00$. (h) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.90$, $\epsilon = 0.10$. (i) $\frac{\tilde{\sigma}^2}{\sigma^2} = 0.90$, $\epsilon = 0.01$.



(j) $\frac{\tilde{\sigma}^2}{\sigma^2} = 1.00$, $\epsilon = 1.00$. (k) $\frac{\tilde{\sigma}^2}{\sigma^2} = 1.00$, $\epsilon = 0.10$. (l) $\frac{\tilde{\sigma}^2}{\sigma^2} = 1.00$, $\epsilon = 0.01$.

Figure 2. The adaptively restored images, found using our scheme given in Table 2. Compare these results to the non-adaptive) restored image given in Figure 1(b). Table 3 gives the ISNR for each of the restored images.

where

$$
\begin{aligned}
u_x(i+\tfrac{1}{2},j) &\equiv \tfrac{\partial u(x,y)}{\partial x}\big|_{(x,y)=(x_{i+\frac{1}{2}},y_j)},\\
u_y(i+\tfrac{1}{2},j) &\equiv \tfrac{\partial u(x,y)}{\partial y}\big|_{(x,y)=(x_{i+\frac{1}{2}},y_j)},\\
u_x(i,j+\tfrac{1}{2}) &\equiv \tfrac{\partial u(x,y)}{\partial x}\big|_{(x,y)=(x_i,y_{j+\frac{1}{2}})},\\
u_y(i,j+\tfrac{1}{2}) &\equiv \tfrac{\partial u(x,y)}{\partial y}\big|_{(x,y)=(x_i,y_{j+\frac{1}{2}})}.
\end{aligned}
\tag{8}
$$

It is easy to see that the first and fourth terms in (8) should be discretized as

$$
\begin{aligned}
u_x(i+\tfrac{1}{2},j) &= u_{i+1,j}-u_{i,j},\\
u_y(i,j+\tfrac{1}{2}) &= u_{i,j+1}-u_{i,j}.
\end{aligned}
$$

One natural approach to discretizing the second and third terms in (8) would be

$$
\begin{aligned}
u_y(i+\tfrac{1}{2},j) &= \tfrac{1}{4}(u_{i,j+1}+u_{i+1,j+1}-u_{i,j-1}-u_{i+1,j-1}),\\
u_x(i,j+\tfrac{1}{2}) &= \tfrac{1}{4}(u_{i+1,j}+u_{i+1,j+1}-u_{i-1,j}-u_{i-1,j+1}).
\end{aligned}
$$

In this paper we use the *minmod* scheme[1] in order to better preserve boundaries of image features:

$$
\begin{aligned}
u_y(i+\tfrac{1}{2},j) &= minmod[\,\tfrac{1}{2}(u_{i,j+1}+u_{i+1,j+1}-u_{i,j}-u_{i+1,j}),\,\tfrac{1}{2}(u_{i,j}+u_{i+1,j}-u_{i,j-1}-u_{i+1,j-1})\,]\\
u_x(i,j+\tfrac{1}{2}) &= minmod[\,\tfrac{1}{2}(u_{i+1,j}+u_{i+1,j+1}-u_{i,j}-u_{i,j+1}),\,\tfrac{1}{2}(u_{i,j}+u_{i,j+1}-u_{i-1,j}-u_{i-1,j+1})\,]
\end{aligned}
$$

where

$$
minmod(a,b)=\frac{sign(a)+sign(b)}{2}\min(|a|,|b|).
$$

The discrete weighted TV functional in R^2 is analogous to the discrete weighted TV functional (6) in R^1:

$$
TV_\omega(u) = \tfrac{1}{2}\{\sum_{i=1}^{n-1}\sum_{j=1}^{n}\omega_{i+\frac{1}{2},j}\sqrt{[u_x(i+\tfrac{1}{2},j)]^2+[u_y(i+\tfrac{1}{2},j)]^2} + \sum_{i=1}^{n}\sum_{j=1}^{n-1}\omega_{i,j+\frac{1}{2}}\sqrt{[u_x(i,j+\tfrac{1}{2})]^2+[u_y(i,j+\tfrac{1}{2})]^2}\,\}
$$

Our adaptive restoration scheme for R^2 is given in Table 4. It is analogous to the adaptive restoration scheme for R^1 given in Table 2.

We point out that as in the R^1 case, the partially restored image found in Step 1 can be used as a first guess in the iterative numerical scheme used to find the adaptively restored image in Step 3, so that the extra computational cost of the adaptive scheme is not significant.

3.2. Results in R^2

We apply our adaptive TV minimizing restoration scheme, as well as the standard scheme for comparison, to five noisy test images: a cross, a triangle, a circle, a square and a hemisphere. The resulting images are found in Figures 3, 4, and 5. Errors between the true image and each of the original (noisy) u_0, standard restored $u_{standard}$, and adaptively restored $u_{adaptive}$ images are given in Table 5.

As demonstrated in Figures 3 and 4 and Table 5, our scheme is superior to standard TV minimizing image restoration for denoising piecewise constant images. At the same time, our adaptive scheme has similar effects as standard TV restoration for denoising smooth images, as demonstrated in Figure 5.

We found that relatively conservative values of ϵ were appropriate in restoring these R^2 test images. For most of the examples we used a value of $\epsilon = 1.0$, except for the case of denoising the cross, in which for this example $\epsilon = 0.1$ gave better results. We conclude that for an image with grayscale values ranging between 0 and 1, a good range would be $0.1 \le \epsilon \le 1.0$, with ϵ being closer to 1 to be more conservative. Of course, if the grayscale range of the image is greater than 0 to 1, ϵ could be chosen larger, in a linearly dependent way. We note that larger values of ϵ also result in a more stable numerical problem.

Algorithm for Adaptive TV Restoration in R^2

1. Find $\tilde{u}$ by solving
   $$\min_{\tilde{u}} TV(\tilde{u}) \quad \text{subject to} \quad \|\tilde{u} - u_0\|^2 = \tilde{\sigma}^2,$$
   where $0 < \frac{\tilde{\sigma}^2}{\sigma^2} \leq 1$.

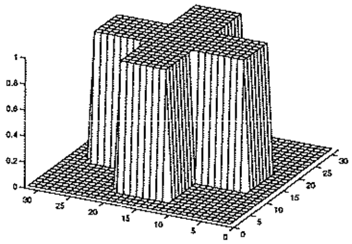
2. For $1 \leq i \leq n-1$, $1 \leq j \leq n$, define
   $$\omega_{i+\frac{1}{2},j} = \frac{1}{|\tilde{u}_{i+1,j} - \tilde{u}_{i,j}| + \epsilon},$$
   and for $1 \leq i \leq n$, $1 \leq j \leq n-1$, define
   $$\omega_{i,j+\frac{1}{2}} = \frac{1}{|\tilde{u}_{i,j+1} - \tilde{u}_{i,j}| + \epsilon},$$
   where $\epsilon > 0$.

3. Find the adaptively restored image u by solving
   $$\min_{u} TV_\omega(u) \quad \text{subject to} \quad \|u - u_0\|^2 = \sigma^2.$$

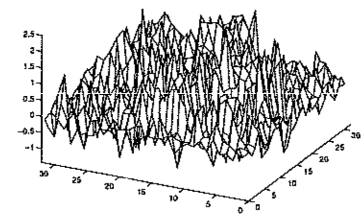
Table 4. Adaptive TV minimizing image restoration scheme in R^2.

Image	ISNR, $u_{standard}$	ISNR, $u_{adaptive}$	ϵ
Cross	8.00 dB	25.80 dB	0.1
Triangle	12.75 dB	19.17 dB	1.0
Circle	13.75 dB	17.35 dB	1.0
Square	9.73 dB	17.12 dB	1.0
Hemisphere	0.39 dB	0.34 dB	1.0

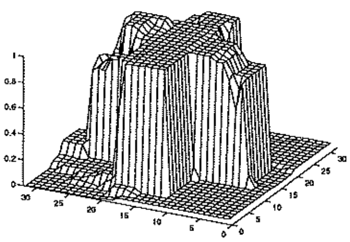
Table 5. The improved SNR for both the standard (non-adaptive) $u_{standard}$ and adaptively restored $u_{adaptive}$ images, which are shown in Figures 3, 4 and 5.



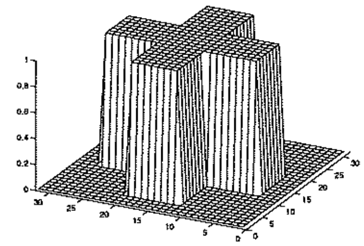
(a) True Image.



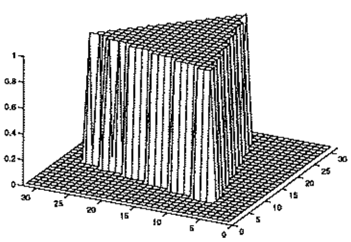
(b) Noisy image, SNR = 0 dB.



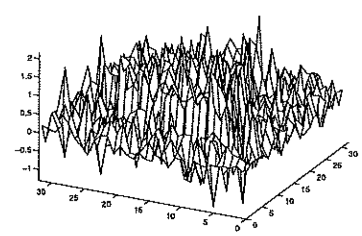
(c) Non-adaptively restored.



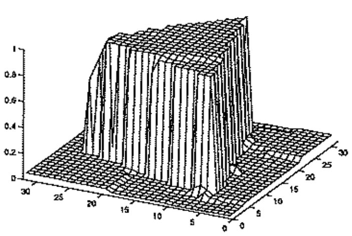
(d) Adaptively restored using $\frac{\tilde{\sigma}^2}{\sigma^2} = 1.0$, $\epsilon = 0.1$.



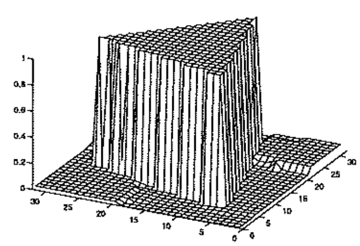
(e) True Image.



(f) Noisy image, SNR = 0 dB.

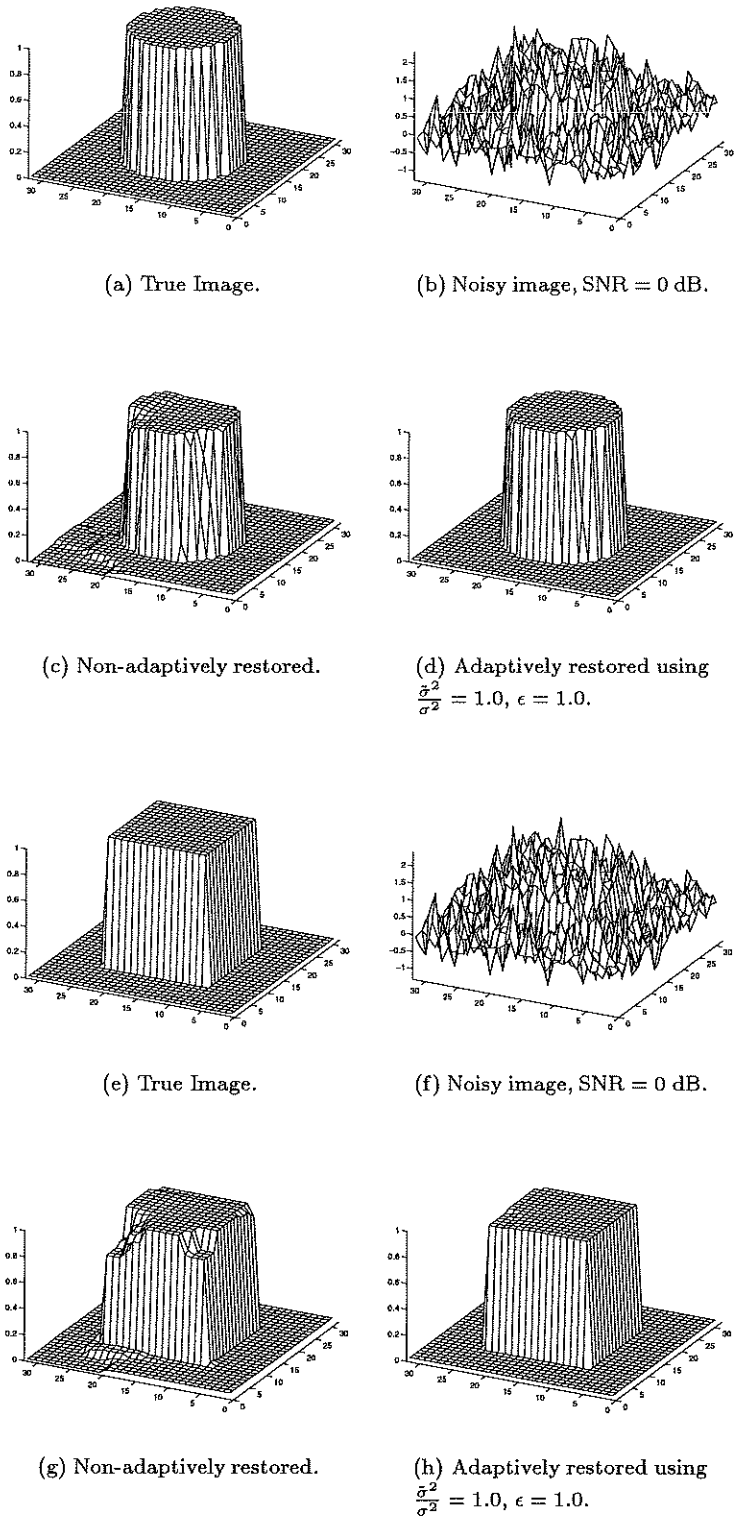


(g) Non-adaptively restored.



(h) Adaptively restored using $\frac{\tilde{\sigma}^2}{\sigma^2} = 1.0$, $\epsilon = 1.0$.

Figure 3. The results of our adaptive restoration scheme (Table 4) are given in (d) and (h), and can be compared to the results of the non-adaptive scheme in (c) and (g). Table 5 gives the ISNR for each of the restored images.



(a) True Image.

(b) Noisy image, SNR = 0 dB.

(c) Non-adaptively restored.

(d) Adaptively restored using $\frac{\tilde{\sigma}^2}{\sigma^2} = 1.0$, $\epsilon = 1.0$.

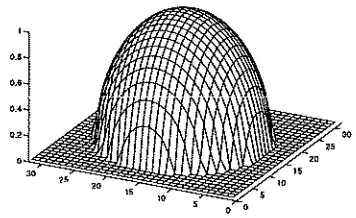
(e) True Image.

(f) Noisy image, SNR = 0 dB.

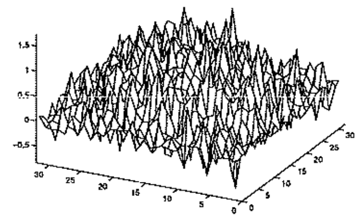
(g) Non-adaptively restored.

(h) Adaptively restored using $\frac{\tilde{\sigma}^2}{\sigma^2} = 1.0$, $\epsilon = 1.0$.

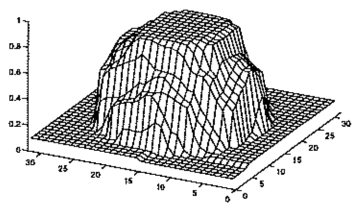
Figure 4. The results of our adaptive restoration scheme (Table 4) are given in (d) and (h), and can be compared to the results of the non-adaptive scheme in (c) and (g). Table 5 gives the ISNR for each of the restored images.



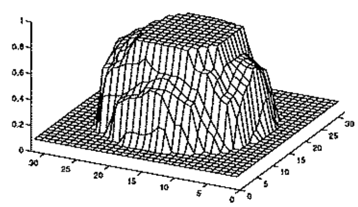
(a) True Image.



(b) Noisy image, SNR = 3 dB.



(c) Non-adaptively restored.



(d) Adaptively restored using $\frac{\hat{\sigma}^2}{\sigma^2} = 1.0$, $\epsilon = 1.0$.

Figure 5. The results of our adaptive restoration scheme (Table 4) are given in (d), and can be compared to the results of the non-adaptive scheme in (c). For smooth images our scheme is quite similar to the non-adaptive scheme for appropriate choices of ϵ. Table 5 gives the ISNR for each of the restored images.

4. SUMMARY

We have given a spatially adaptive total variation (TV) minimizing image restoration scheme, where the adaptivity is realized by weighting the measure of total variation of the image. A spatially varying weighting factor is chosen to be inversely proportional to the likelihood of there being an edge (i.e. discontinuity) between two neighboring pixels. In this paper, the original image is partially restored, then the weighting factor is determined by the size of the jumps between neighboring pixels of this partially restored image. To control the effect of the weighting factor, as well as to improve the stability of the resulting numerical problem, a parameter is chosen (which herein is labelled ϵ). We found that the appropriate value of ϵ should be approximately equal or slightly less than the grayscale range of the image. Our adaptive TV minimzing image restoration scheme proved to be quite effective and superior to standard (non-adaptive) TV minimizing restoration in restoring piecewise constant image features. The adaptive and standard schemes were quite similar in restoring smooth image features. The extra numerical cost of solving the adaptive restoration problem is not great relative to the cost of solving the standard restoring problem.

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