Feature Preserving Lossy Image Compression Using Nonlinear PDE's*

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Abstract

In this report, we propose combining the Total Variation denoising method with the high loss wavelet compression for high noise level images. Numerical experiments show that TV-denoising can bring more wavelet coefficients closer to zero thereby making the compression more efficient while the salient features (edges) of the images can still be retained.

1 Introduction

Image compression techniques have been generally used in image communication and storage. In this report, we consider the possibility of high loss compression of very noisy images. There are many applications such as automatic target recognition (ATR), ATM and surveillance imaging, in which images are taken in a high noise environment. Interesting parts of the images are the shape and location of objects. our goal is that after the high loss compression, the salient features (edges) of the images are still preserved so that the shape and location of the objects can be well recognized.

Wavelets have become the most important tool for image compression in recent years. Through wavelet transformation, the correlation on image pixels can be decoupled, and the energy can be concentrated in a small set of coefficients. Moreover, wavelet coefficients have hierarchical multiresolution structures. These properties of wavelet decomposition make it an effective tool in image coding. For a recent survey, see [7]. For noiseless images or images without lots of noise, wavelet transform based compression methods can give high ratio compression

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and good image quality. Wavelets are also used as denoising tool in image processing. Typically, an image can be denoised and compressed simultaneously by truncating a certain range (usually the high frequency range) of wavelet coefficients. For instance, MATLAB/wavelet/wavelet toolbox uses such techniques to denoise and compress images. However for very noisy images, denoising and compression using wavelets band pass filters or thresholding may not work well since the features of the images may be lost while removing the noise. The reason is that wavelet decomposition cannot efficiently distinguish edges from noise, because they both generate high frequencies. In the present work, we advocate a feature-retaining denoising method, followed by wavelet compression to get a high ratio compression which still keeps the features. In particular, we consider the total variation (TV) denoising method which can smooth out the high frequency oscillation while keeping the edges in the high noisy level images. Total variation is one of the most successful of a class of recently emerging image processing methods using nonlinear partial differential equations; for a recent survey, see [1].

Our numerical examples show that TV-denoising can bring more wavelet coefficients closer to zero than that of the original noisy image, thus making the compression more efficient. Coefficients generated by edges are still relatively big and therefore well retained. In our numerical experiments, we also note that TV-denoised images are not sensitive to the thresholding. In other words, if we only remember the largest m wavelet coefficients and set the rest to zero, compressed TV-denoised images keep their features for a large range of m while without TV-denoising the compressed noisy images are very sensitive to the selection of m. They either lose the features for relatively small m or cannot remove the noise when a bigger m is chosen.

The main focus of our present work is on the denoising and the transformation aspects of the compression process. We do not consider quantization and coding here. The report is organized as follows. In the next section, we briefly review wavelet compression. In section 3, we state the basic idea of TV denoising. And finally, we give our preliminary numerical results in section 4.

2 Wavelets compression

There is a vast literature on wavelet image compression, see [7] and the references listed in that paper. Wavelet transform can decouple the correlation among the modes, thereby concentrating the energy in a few coefficients. That means there are a lot of near zero elements in the wavelet coefficients. Also, wavelet decompositions have a multiresolution framework. Most of the wavelet compression methods are based on these properties of wavelet coefficient, such as the famous 'zerotree' method [13]. Essentially, the near zero coefficients can be treated as zeros and do not need to be memorized and therefore, compression can be achieved. The more zero coefficients, the higher compression ratio can be obtained.

Two general approaches are commonly used in setting the wavelet coefficients to zero. The first one is low pass filter. First, represent the image u using wavelet basis, i.e

$$c = W(u),$$

where W is the wavelet transform and c is the wavelet coefficients of u. Since the high frequency part essentially corresponds to the noise, one sets all coefficients of these modes to zero and just

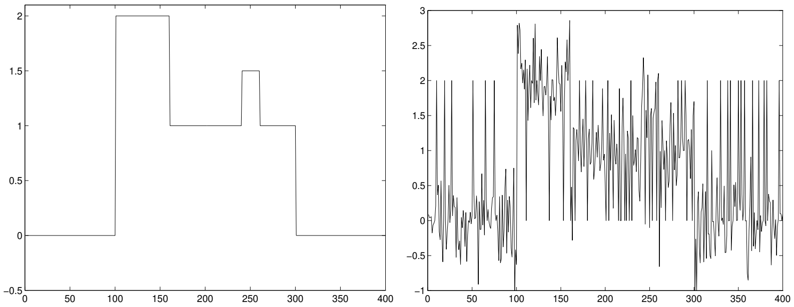


Figure 1. Original Image (left) and Noisy Image (right)

memorize the low frequency part, see [10]. Unfortunately, this approach can sometimes destroy essential features of the image if these features are also in the high frequency range, e.g. edges.

The second approach is thresholding, which is considered as a better feature retaining approach [9], [11]. In the simplest case, the following thresholding function:

$$T(\epsilon, x) = \begin{cases} 0, & \text{if } |x| \leq \epsilon \\ x, & \text{otherwise} \end{cases}$$

can be applied to the wavelet coefficients c. All the coefficients whose absolute values are less than ϵ are set to zero. This simple approach is called 'hard thresholding'. Donoho also proposed a variant called 'soft thresholding' in [8] and softwares for such thresholding methods can be found in toolbox MATLAB/wavelet/wavelet.

For noiseless images or low noisy level images, wavelet thresholding compression can get a high compression ratio while still keeping the features of the image. But for very noisy images, wavelet transform cannot keep features as well as getting high compression ratio. Figures 1 and 2 is an example of hard thresholding wavelet compression. Figure 1 are the original image (left) and the noisy image (right). In Figure 2, the left one is the compressed image which keep the largest $m = 50$ wavelet coefficients, the right one is the compressed image with keeping the largest $m = 20$ coefficients. As we can see, for $m = 50$, the noise is still significantly present in the compressed image, whereas for the case $m = 20$, although the noise is removed almost completely, some features (e.g. the "bump" around pixel number 250) of the image are also lost. This illustrates that hard thresholding may not work well for the high noise level images.

3 Total Variation Denoising

TV denoising considers the following noise model:

$$u_0(x, y) = u(x, y) + \eta(x, y) \tag{1}$$

where $\eta(x, y)$ is a Gaussian white noise, $u_0(x, y)$ is the observed intensity function of the image and $u(x, y)$ the original image. The objective of TV denoising is to compute a denoised image by

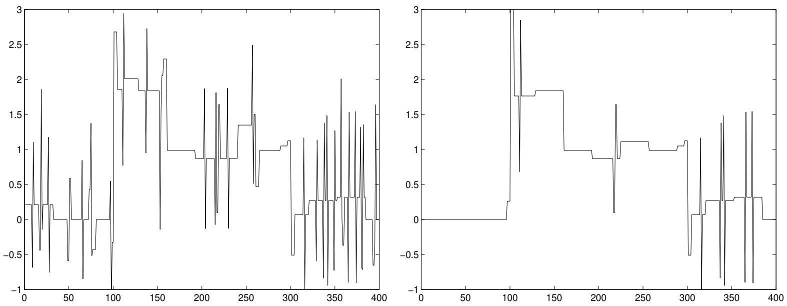
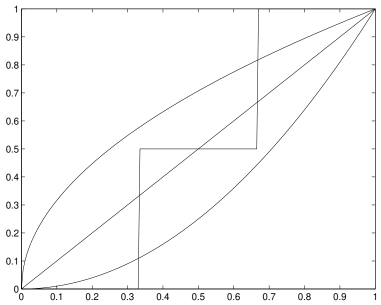


Figure 2. Compressed Image
Left: The largest $m = 50$ coefficients are kept
Right: The largest $m = 20$ coefficients are kept



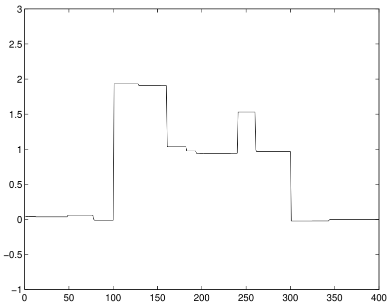


Figure 3 (left). The four functions in graph have the same TV norm.
Figure 4 (right). TV-norm Denoised Image of Noisy Image in Figure 2

solving the following constrained minimization problem proposed by Rudin, Osher and Fatemi [12]:

$$\min_u TV(u) \equiv \int_\Omega |\nabla u| dx, \tag{2}$$

subject to

$$||u - u_0||^2 = \sigma^2. \tag{3}$$

Here Ω is a bounded convex region in $\mathbb{R}^d$, $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^d$, $||\cdot||$ denotes the norm in $\mathbf{L}^2(\Omega)$ and σ is the standard deviation of the noise $\eta(x, y)$.

Intuitively speaking, the TV norm measures the jumps of the images, no matter whether they are continuous or discontinuous. An important feature of TV norm denoising is that it does not penalize discontinuities in the image. For example, in Figure 3, the four functions have the same TV norm. Therefore, by solving the minimization problem (2) and (3), the data automatically determines whether a smooth function or a discontinuous function best models the restored image. It is important to note that it doesn't artificially over sharpen the edges. Hence, the most significant advantage of TV denoising is that while smoothing out the high frequency oscillations (corresponding to noise) in the image, it preserves the edges systematically. For instance, Figure 4 shows the TV-denoising result of the 1-dimensional image in Figure 2 (left).

Here we use the TV-denoising algorithm described in [14]. It is clearly seen that the features (edges) of the image are kept. More comparisons of TV denoising method with other restoration methods are shown in Figure 5.

There are now many papers discussing TV denoising; see for example [2], [3], [4], [5], [6], [17], [15], [16] and [18].

4 Wavelet Compression of TV Denoised Images

We consider 1-dimensional and 2-dimensional high noise level images in this preliminary numerical test. Our approach is the following.

We first do a TV-denoising for the noisy image, then compute the wavelet (Haar in this paper) transform of the denoised image. After this we apply the hard thresholding function to the wavelet coefficients. In the examples, we compare the TV-denoised and wavelet compressed images with the images obtained by denoising and compressing directly using wavelet thresholding.

We must mention that there is a computational cost associated with TV-denoising, but it may be worth it, e.g. in off line compression.

In the following examples, the thresholding is done in following way: given a certain positive integer m, we keep the largest m wavelet coefficients (in absolute values) unchanged and set the others to zeros. We define PNT to be the percentage of wavelet coefficients kept:

$$PNT = \frac{(number \quad of \quad nonzero \quad coefficient)}{total \quad number \quad of \quad coefficient} \times 100.$$

In the examples, we also compare the thresholding tolerance ϵ, which indicates that all the coefficients whose absolute values are less than ϵ are set to zero, between the direct wavelet denoising and compression images and TV-denoising and compression images. DTOL stands for the tolerance of direct wavelet denoising and compression images while TTOL indicates the tolerance of TV-denoising and wavelet compression images.

The first example is testing the TV denoised image in Figure 4. We choose different m's, compute the PNT numbers and compare the images and tolerances.

In table 1, we compare the hard thresholding tolerance ϵ values of the 'Haar' wavelet coefficients with five levels before and after the TV-denoising respectively. We see that for the same m, after TV-denoising, the tolerance TTOL is much smaller than using wavelet directly. In other words, TV-denoising can bring more wavelets coefficients closer to zero.This is because the TV-denoised image is smoother than the noisy image.

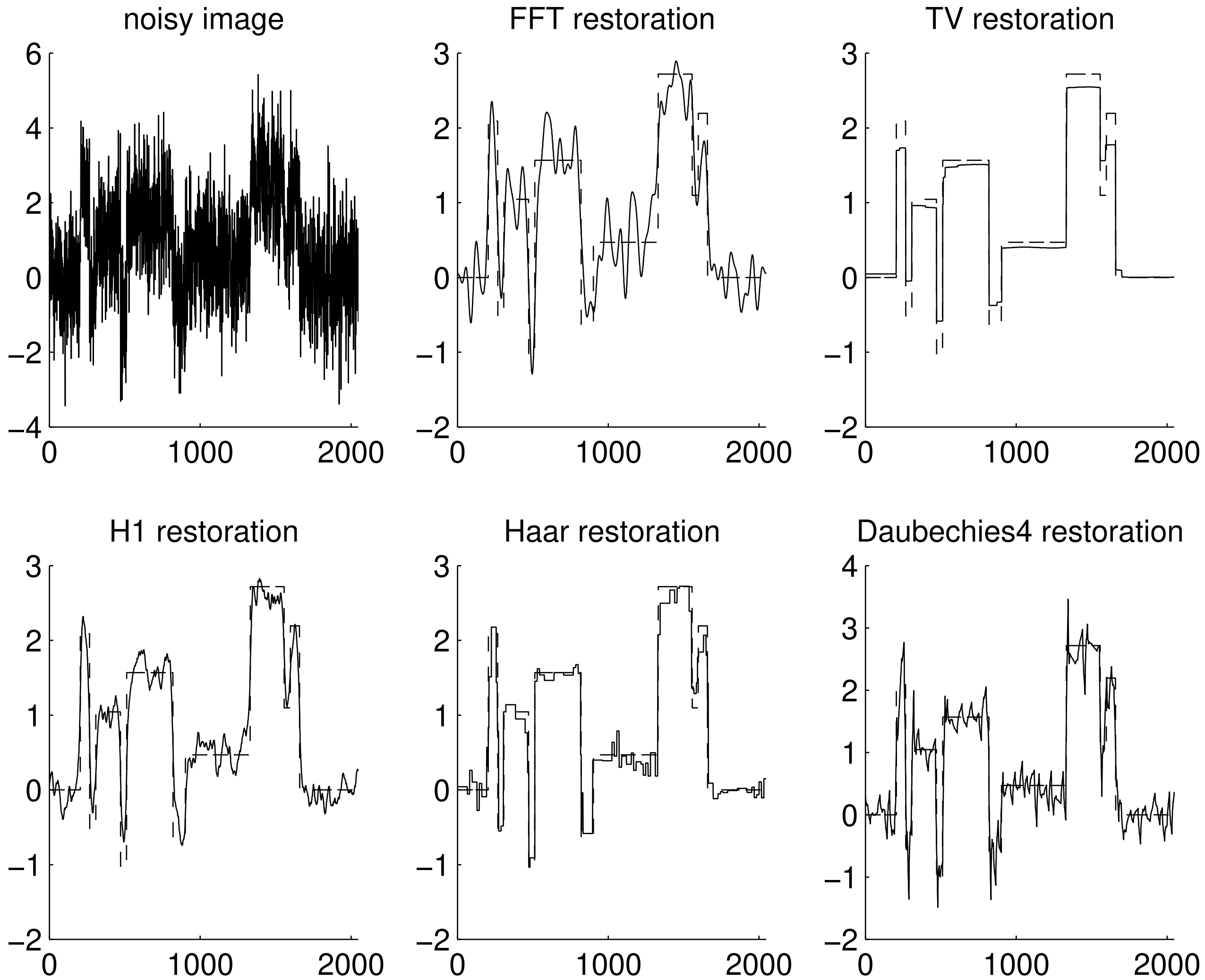


Figure 5, Comparison of TV denoising method with other restoration methods.

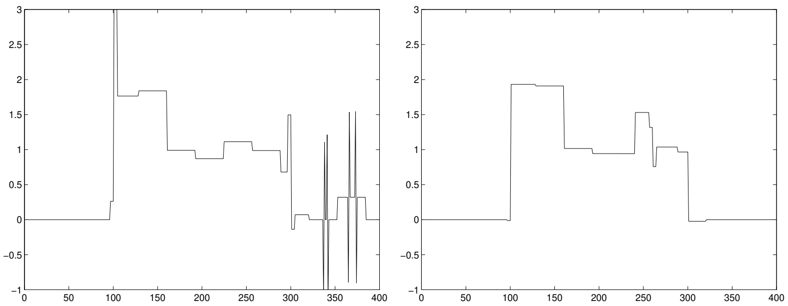


Figure 6. $m = 15$ Compressed Image without TV Denoising (left,$\epsilon = 1.41$) and Compressed Image with Denoising(right, $\epsilon = 0.07$).

m	PNT	DTOL	TTOL
15	3.74	1.57	0.79
25	6.23	1.41	0.07
35	8.73	1.34	0.02
45	11.22	1.22	0.006
55	13.71	1.14	4.4e-4
65	16.21	1.05	1.0e-4
100	24.94	0.83	3.5e-5

Table 1. Tolerance values ϵ before the total variation denoising and after it respectively for Example 2.

Figures 6 to 9 show the compressed images with $m = 15, 35, 55$, and 100 respectively. The left pictures are the compressed images without total variation denoising. The right are the compressed images with total variation denoising. The dash lines in the pictures are the original image. We notice that, for the images without TV-denoising, if the noise is smoothed out almost completely, the features of images are lost already (e.g. the "bump" around pixel 250); see Figure 6. On the other hand, the images without TV-denoising cannot eliminate the noise if we want to keep the salient features of the images. For the TV-denoising images, they maintain the features for a wide range of m, see Figure 7 to 9. It begins to lose the features only after m becomes less than 15. This illustrates that TV-denoising compressed images are not sensitive to the choice of m.

We next consider a 2-dimensional 64×64 image, The noise level σ of the image is 0.4 and the corresponding SNR is -4.08dB. In Figure 10 and 11, the original image, the noisy image and the TV denoised image are shown respectively.

In Table 2, the PNT numbers are presented for different available space m, as well as the tolerances DTOL and TTOL. For the same m, TTOL is much smaller than DTOL but the difference is not as much as in 1-dimensional case. This may be due to two reasons. First, in 2-D, there are more edges in the images and therefore more coefficients are needed to represent them. Second, we note that in the 2-D case, the PNT numbers are still small compared to the 1-D case even when m is 300.

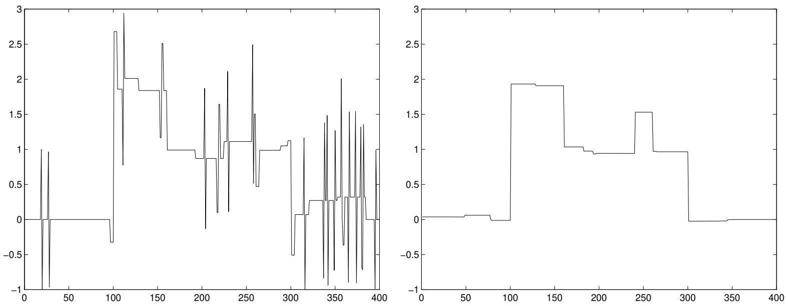


Figure 7. $m = 35$. Compressed Image without TV Denoising (left,$\epsilon = 1.34$) and Compressed Image with Denoising(right, $\epsilon = 0.02$).

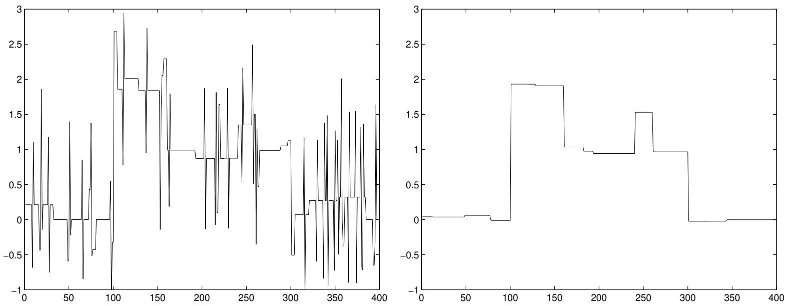


Figure 8. $m = 55$. Compressed Image without TV Denoising (left,$\epsilon = 1.13$) and Compressed Image with Denoising(right, $\epsilon = 4.4e - 4$).

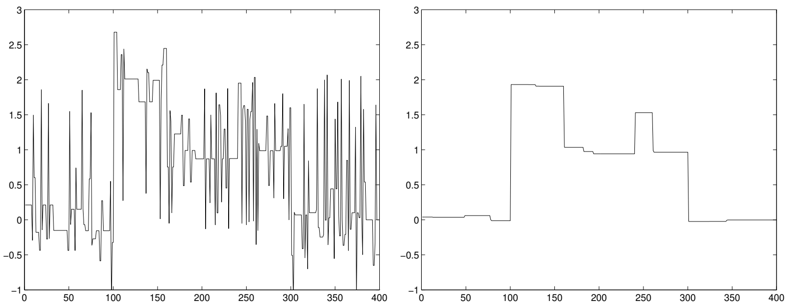


Figure 9. $m = 100$. Compressed Image without TV Denoising (left,$\epsilon = 0.83$) and Compressed Image with Denoising(right, $\epsilon = 3.5e - 5$).

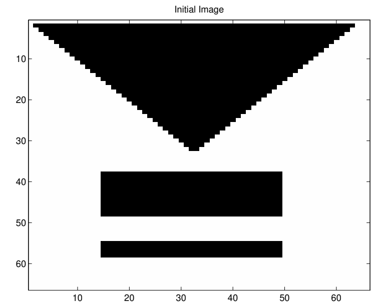


Figure 10. Original Image.

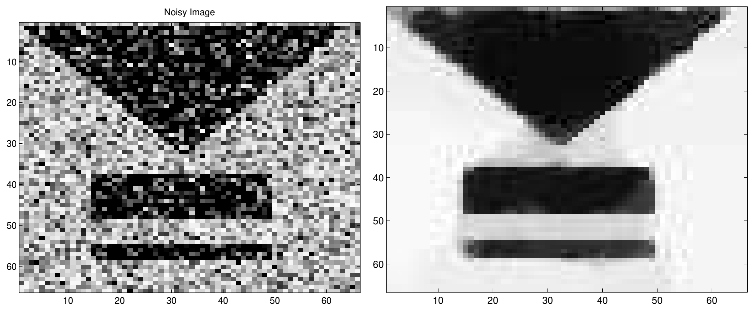


Figure 11. Noisy Image (left) and Denoised Image (right)

m	PNT	DTOL	TTOL
50	1.11	2.14	1.42
75	1.67	1.63	0.95
100	2.23	1.29	0.70
150	3.34	1.09	0.52
200	4.46	0.99	0.36
300	6.68	0.84	0.23

Table 2, The tolerance values ϵ for Example 2.

Figures 12 to 14 show the thresholded images before and after the TV-denoising respectively. The threshold parameter m is $75, 100$, and 150 respectively. From the table and figures, we note that for the 2-dimensional case, the results are qualitatively similar to 1-dimensional case.

The next figure (Figure 15) is a cross section of Figure 13. In the figure, the dotted line is the cross section of original image, the dash line is the compressed image without TV denoising while the solid line is the compressed image with TV denoising. We notice that for the undenoised compressed image, there are Gibbs-like oscillations at the edge areas while there is no oscillation for the TV denoised compressed image.

To avoid these oscillations at the edge areas, a possible way of wavelet denoising and compression is the following: First truncate the wavelet coefficients to reach the purpose of denoising, then transform the image back to physical space and use artificial edge enhancement to elimi-

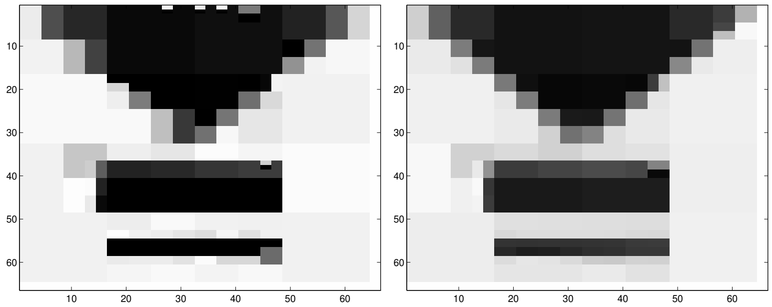


Figure 12. $m = 75$. Compressed Image without TV-denoising(left, $\epsilon = 1.63$). and that with TV-denoising (right, $\epsilon = 0.95$).

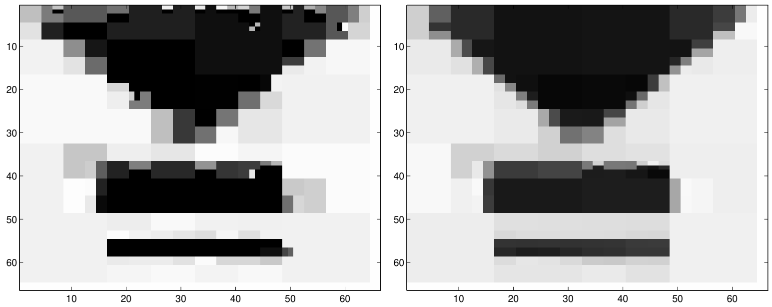


Figure 13. $m = 100$. Compressed Image without TV-denoising(left, $\epsilon = 1.29$). and that with TV-denoising (right, $\epsilon = 0.70$).

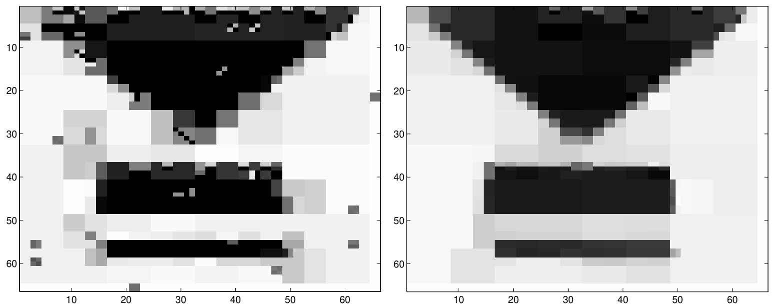


Figure 14. $m = 150$. Compressed Image without TV-denoising(left, $\epsilon = 1.09$). and that with TV-denoising (right, $\epsilon = 0.52$).

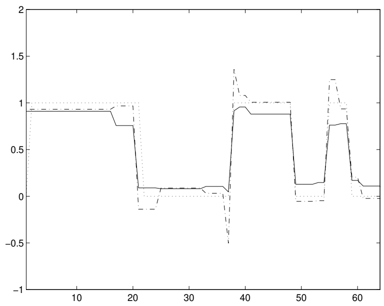


Figure 15. Cross Section of Compressed Image without TV denoising(dash line) and that with TV denoising (solid line),$m = 100$.

nate the oscillations at the edges, and then, transform to wavelet basis again and truncate the coefficients to arrive the end of compression. Comparing this with TV-denoising compression, we believe that the latter can more systematically avoid these oscillations.

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