A sample day of class: Fractal curves

Goals:

- Organically introduce the concepts of infinite sequences and infinite series in accessible fashion via perimeters and areas of fractal curves.
- Engage students of all backgrounds with the likely exotic and interesting concept of fractals.
- Emphasize pattern recognition and exploration of ideas over notation and formulas.

Outline of the class:

- Notes for instructor presentation 1: 20 mins
  - The *coastline paradox*: Can we measure the length of a coastline precisely?[1] Depending on the size of the ruler used, we get wildly different answers. What's up with this? To understand this mathematically, we consider *fractal curves*.
  - Example: one side of the Koch snowflake. Guide the students through computing the length of the first few iterations of the snowflake and establishing the pattern to identify a formula for the perimeter at stage n.
  - Use this to introduce the concept of a *sequence* as an infinite list of numbers. What happens if we continue the snowflake construction forever? Examine the numbers as n gets larger to conclude that the perimeter is infinite.
- Worksheet part 1: 30 mins
  - Example of another fractal curve (e.g. with squares). Prompts and problems that walk through the same pattern recognition process.
  - A prompt that asks the students to come up with their own fractal-like curves. Be creative, there are no rules! (For example: start with a straight line, cut out middle third and append a semicircle. Repeat with each straight segment. Or, give them names of other fractal curves to Google for inspiration.) Can you find the perimeter patterns? Can you build a fractal-like curve that seems to have finite perimeter?
- Break: 5-10 mins

[1] Reference: Mandelbrot 67′: How long is the coastline of Britain?

- Notes for instructor presentation 2: 5 mins
  - Draw the triangle version of Koch snowflake. Observe that we can compute areas of the enclosed shape at each stage.
- Worksheet part 2: 25 mins
  - Prompts that walk them through the pattern recognition of the area at the nth stage.
- Notes for instructor closing remarks: 1 min
  - Summarize the area pattern recognition problem with the notion of an infinite sum.
  - It turns out the area is finite. Thus, we have a curve with infinite length that encloses a finite area.
  - If time permits, allude to other things like Gabriel's horn. (This can potentially be a topic as well, if the students liked this session).

Other notes:

- There will be students with a wide variety of backgrounds in the Bridge program. For stronger students with prior calculus exposure, here are potential additions to the worksheet to keep them engaged:
  - Writing pseudocode, or perhaps even real code, for a program that can input n and output the area and the perimeter at the nth step of the construction of the Koch snowflake.
  - Walking through a proof of the geometric series summation formula.
  - Deriving the nth term divergence test.
  - Plotting partial sums of various series with positive and negative terms.
  - Experimenting, numerically, with a conditionally convergent series, to see that the order in which we add the terms may end up mattering. (Caution! This might cause a lot of confusion).
  - Introducing the idea of *fractal dimension*.