# GEOMETRY/TOPOLOGY QUAL GUIDELINES

The Geometry/Topology (G/T) qualifying exam corresponds to the course sequence Math 225ABC.

**Format and passing score.** The G/T qual consists of 10 problems, each worth 10 points, for a maximum score of 100. In order to pass the exam the student must usually:

(1) receive at least 60/100; AND
(2) either (i) solve 6 problems more or less completely (i.e., receive at least 8/10 on 6 problems) OR (ii) solve 5 problems more or less completely and receive at least 5/10 on two other problems.

**Topics covered on the exam.**

**1. Smooth manifolds (Math 225A)**

(1) Smooth manifolds and smooth maps.
(2) Inverse and implicit function theorems, submersions, immersions, submanifolds.
(3) Tangent and cotangent bundles, vector bundles.
(4) Differential forms, exterior differentiation, and Lie derivatives.
(5) Integration, Stokes' theorem, de Rham cohomology, and computations using the Meyer-Vietoris sequences.
(6) Vector fields, integral curves, distributions, Frobenius' theorem.

*References.*

- Notes by Peter Petersen (http://www.math.ucla.edu/ petersen).
- Notes by Ko Honda (http://www.math.ucla.edu/ honda).
- Lee, *Introduction to smooth manifolds*.
- Spivak, *A comprehensible introduction to differential geometry*.
- Warner, *Foundations of differentiable manifolds and Lie groups*.

**2. Differential topology (Math 225B)**

(1) Sard's theorem and transversality, Whitney embedding theorem, tubular neighborhoods.
(2) Intersection theory, degree, vector fields and Poincaré-Hopf theorem, Lefschetz fixed point formula.
(3) Compactly supported cohomology, Poincaré duality, Thom isomorphism, and the Künneth theorem from the point of view of de Rham theory.
(4) Applications: homotopy types of self-maps of tori and spheres (Hopf degree theorem); Lefschetz numbers of self-maps of spheres, real and complex projective spaces, and tori.

*References.*

- Guillemin and Pollack, *Differential topology* (but the perspective is a bit elementary since manifolds are assumed to be already embedded in Euclidean space).
- Bott and Tu, *Differential forms in algebraic topology*, Chapter 1.
- Hirsch, *Differential topology*.
- Benedetti, *Lectures on differential topology* (much more advanced than necessary, but has a careful treatment of many constructions).

**3. Algebraic topology (Math 225C)**

(1) Homotopy theory: fundamental group, covering spaces, Van Kampen's theorem.
(2) Homology theory: singular homology, simplicial homology, homotopy invariance, relative homology, excision and Mayer-Vietoris, functoriality, relationship to the fundamental group, calculations with CW complexes.
(3) Cohomology theory: singular cohomology, universal coefficient theorem, cup products. (Singular cohomology is usually not covered thoroughly, so being able to use the universal coefficient theorem and cup products is sufficient.)

*References.*

- Hatcher, *Algebraic topology*, Chapters 1–3.