UCLA College | Physical Sciences Mathematics

Optimization / Numerical Linear Algebra

The ONLA qualifying exam is the shared responsibility of the applied group. It corresponds to the graduate courses 273A and 270BC in Math. The following sections describe the prerequisites from undergraduate study as well as the graduate-level topics for the exam. Students are encouraged to study these topics broadly. The exam typically consists of 8 questions, with 2-3 questions arising from the material in each of the three courses previously mentioned. An exam with 2 out of 3 essentially complete questions (a score of 9 or 10 out of 10) in each of the three course areas will usually lead to a Pass (and for areas with only 2 questions, something like 1 essentially complete and 1 demonstrating a majority of completion (7-10 score)). Scores that lie close to this threshold or that depend heavily on partial credit are assessed on a case-by-case basis. Note that an excellent score in one of the three areas but a poor score in another is usually not enough for a Pass.

Prerequisites

The ONLA qualifying exam and the corresponding courses will assume a good working knowledge of linear algebra and proof techniques at about the undergraduate honors level. This is taken to include all topics for the Basic Exam, and in particular the following material from linear algebra: systems of linear equations, matrix algebra, least-squares methods, determinants, eigenvalues and eigenvectors, matrix diagonalization, matrix factorization, invertible matrices, orthogonality, rank-nullity theorem, Gram-Schmidt process, QR factorization.

These prerequisites will generally not be taught again in the graduate courses, with the exception of a more in depth study of Gram-Schmidt and QR factorization. These prerequisite topics and those above can be found in e.g. *O. Bretscher, Linear Algebra, 5th Ed., Prentice Hall.* Students who still need to study these topics are encouraged to approach an instructor of the corresponding course, or any other member of the applied group for support or guidance. In the past, many students in this position have benefited from studying these topics as a group. It is completely respectable and understandable that students enter graduate school with varying levels of experience and knowledge, and this is in no way a reflection of their intellect or self-worth. The department and applied group is always hopeful that every student will be successful in the program, and offers support and guidance to help.

Graduate level topics

Beyond the prerequisites described above, the following topics are relevant to the exam in the following way: The background knowledge relevant to the exam must not exceed this list, and these topics must be covered in the relevant courses. The instructor of such courses may cover additional topics at their discretion.

A note – the statements above do not mean that qualifying exam questions are generally comparable to final exam questions from the corresponding courses. On the whole, qualifying exam questions require more of a synthesis of ideas from multiple areas covered in these courses. Students studying for the exam are encouraged to study the topics thematically and also practice on both course problems as well as past exam problems. In fact, writing one's own problems is also a great way to study. Past exam problems are not necessarily representative of future exams, so these should serve as only one avenue for study and practice.

Course material: Mathematics 273A and Mathematics 270BC.

Optimization Topics (typically covered in 273A): properties of convex functions and convex sets; subgradients and gradients; proximal operators; convex duality; Lagrangian and augmented Lagrangian; gradient descent method, proximal-gradient method, ADMM; forward-backward and Douglas-Rachford splitting methods; stochastic gradient method; coordinate gradient method. The typical applications of these abstract methods. Understanding the tradeoffs between different methods induced by problem structure.

Numerical Linear Algebra Topics (typically covered in 270BC): Direct, fast, and iterative algorithms, singular value decomposition, regularization, eigenvalue problems, QR Factorizations, Givens and Householder rotations, least squares solvers, conditioning and stability, Gaussian elimination, Cholesky factorization, Hessenberg reduction, QR algorithm, classical iterations for linear systems, Arnoldi method, Lanczos method, GMRES, conjugate gradient methods, power method, orthogonalization methods, subspace iteration, discrete transform methods.

References

Note these references do not correspond exactly to the outlined topics above, but many good texts cover these topics. The texts used for the relevant courses are chosen by the individual instructors. Some popular and useful texts are listed here.

- Golub and Van Loan, Matrix Computations 4th edition, Johns Hopkins Press
- Trefethen and Bau, Numerical Linear Algebra
- Demmel, Applied Numerical Linear Algebra, SIAM
- Boyd and Vandenberghe, Convex Optimization, Chapters 2-5
- Bertsekas, Convex Optimization Theory
- Boyd, Parikh, Chu, Peleato, Eckstein, Distributed Optimization and Statistical Learning via the Alternating Direction Method of Multipliers.
- Leon Bottou, Large-Scale Machine Learning with Stochastic Gradient Descent.