
VICON: Vision In-Context Operator Networks for Multi-Physics Fluid Dynamics Prediction

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Abstract

In-Context Operator Networks (ICONS) are models that learn operators across different types of PDEs using a few-shot, in-context approach. Although they show successful generalization to various PDEs, existing methods treat each data point as a single token, and suffer from computational inefficiency when processing dense data, limiting their application in higher spatial dimensions. In this work, we propose *Vision In-Context Operator Networks* (VICON), incorporating a vision transformer architecture that efficiently processes 2D functions through patch-wise operations. We evaluated our method on three fluid dynamics datasets, demonstrating both superior performance (reducing the rescaled L^2 error by 40% and 61.6% for two benchmark datasets for compressible flows, respectively) and computational efficiency (requiring only one-third of the inference time per frame) in long-term rollout predictions compared to the current state-of-the-art sequence-to-sequence model with fixed timestep prediction: Multiple Physics Pretraining (MPP). Compared to MPP, our method preserves the benefits of in-context operator learning, enabling flexible context formation when dealing with insufficient frame counts or varying timestep values.

1. Introduction

Machine learning has recently emerged as a powerful tool for solving Partial Differential Equations (PDEs). Traditional approaches primarily operate on discrete representations, with Convolutional Neural Networks (CNNs) excelling at processing regular grids (Gupta & Brandstetter, 2022) and Graph Neural Networks (GNNs) handling un-

structured meshes (Sanchez-Gonzalez et al., 2020; Lino et al., 2022; Cao et al., 2023). Although recent developments have incorporated transformer architectures to capture global dependencies (Cao, 2021), these discrete approaches often face two key limitations: they lack explicit PDE context information and are confined to operating on fixed or similar topologies.

Several methods have emerged to address these limitations. The Deep Galerkin Method (DGM) (Sirignano & Spiliopoulos, 2018), Deep Ritz Method (DRM) (E & Yu, 2018), and Physics-Informed Neural Networks (PINN) (Raissi et al., 2019) incorporate PDE constraints in the model loss terms. The model, once trained, becomes a surrogate solution that can be queried at any location in the domain. However, these methods typically require complete retraining when initial or boundary conditions change. This limitation led to the development of “operator learning,” where neural networks map input functions to output functions, such as from initial conditions to later-time solutions. This capability was first demonstrated using shallow neural networks (Chen & Chen, 1995b;a), followed by specialized architectures such as the widely-adopted Fourier Neural Operator (FNO) (Li et al., 2021; Kovachki et al., 2023), Deep Operator Network (DeepONet) (Lu et al., 2021), and recent attempts to incorporate transformers as operator learners (Li et al., 2022).

Although the aforementioned methods excel at learning individual, single operators, they are not able to generalize across different operators. This limitation necessitates costly retraining for new equation parameters or predicting with longer time horizons. Inspired by the success of large language models (LLMs) in multi-domain generation, recent work has explored “PDE foundation models”. Common approaches employ pre-training and fine-tuning strategies (Chen et al., 2024; Herde et al., 2024; Zhang et al., 2024c), though these require additional deployment resources. To achieve zero-shot PDE foundation models, additional information is often needed, for example, the PROSE approach (Liu et al., 2024b;a; Sun et al., 2024a; Jollie et al., 2024) encodes the governing equations directly through symbolic information. Other approaches achieve zero- or few-shot generalization by using directed graphs to

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represent equations (Ye et al., 2024) or symbol tokenization (Lorsung et al., 2024).

The In-Context Operator Network (ICON) is able to implicitly incorporate PDE system dynamics through input-output function pairs (before-after operators), achieving few-shot generalization without retraining or knowledge of the PDE information. As shown in (Yang et al., 2023a), a single ICON model can handle forward and inverse problems in various ODEs, PDEs, and mean-field control scenarios. However, vanilla ICONs face computational efficiency challenges: representing functions as scattered point tokens results in quadratic computational complexity relative to the number of data points. This scalability limitation has confined previous ICON applications to simple 1D problems, with only one instance of a 2D case using sparse data points. In this work, we propose a novel approach that leverages vision transformers for in-context operator learning of 2D functions. Our method represents 2D functions as grid-based images and divides them into sequences of patches. To enable “next function prediction” like in the original ICONs, we extend beyond classic vision transformers, where they process the sequence of patches of a single image, to handle sequences of input-output function pairs.

We name our new method *Vision In-Context Operator Networks* (VICON). The model architecture is illustrated in Figure 1. Our main contributions include the following.

- Development of VICON, a vision transformer-based in-context operator network for two-dimensional time-dependent PDEs that maintains the in-context operator learning framework and handles dense data points in higher dimensions.
- We evaluated our model’s performance in forward prediction tasks for different kinds of fluid dynamics system with varying timestep sizes, comprising 879K frames in total with diverse dynamics. Empirical results show better long-term prediction performance and lower computational cost compared to the state-of-the-art competitor: multi-physics pretraining (MPP) (McCabe et al., 2023).
- VICON demonstrates advantages in the flexibility in forming function pairs, compared to strict sequence-to-sequence models, and the ability to adopt flexible inference strategies, such as multistep predictions.

2. Related Works

Operator Learning Operator learning (Chen & Chen, 1995b;a; Li et al., 2021; Lu et al., 2021) addresses the challenge of approximating operators $G : U \rightarrow V$, where U, V are function spaces. These operators usually model solutions of physical systems and differential equations,

representing the map from initial/boundary conditions and parameters to equation solutions. DeepONets (Lu et al., 2021; 2022) use branch and trunk networks to separately process inputs and query points, and have been extended to handle multiple input parametric functions in MIONet (Jin et al., 2022). FNOs (Li et al., 2021) leverage kernel integration approximations for PDE solutions by processing features in Fourier space, where kernel integrations can be efficiently computed through multiplications. These methods have been further extended to incorporate equation information (Wang et al., 2021), multiscale features (Zhang et al., 2024a; Wen et al., 2022), different input/output meshes (Zhang et al., 2023), complex geometries (Li et al., 2024), and distributed learning (Zhang et al., 2024b). The applications of operator learning include weather prediction (Pathak et al., 2022; Jiang et al., 2024), biology (Yin et al., 2024), and uncertainty quantification (Moya et al., 2024). However, these approaches are typically designed to learn a single operator, requiring retraining when encountering different types of PDE or time-step sizes.

PDE Foundation Models Foundation models are large-scale, pre-trained models that serve as versatile building blocks for a wide range of downstream tasks in natural language processing (Brown et al., 2020; Touvron et al., 2023), computer vision (Ramesh et al., 2021), and other AI applications. However, they are not directly applicable to scientific computing tasks such as the discovery of PDEs (Schaeffer, 2017) or computational fluid dynamics (Wang et al., 2024), where high precision is essential. Recent work has explored adapting the principles of the foundation model to PDEs through various approaches. Some methods employ pre-training and fine-tuning strategies (Chen et al., 2024; Herde et al., 2024), though this requires additional computational resources during deployment. Zero-shot PDE foundation models incorporate additional information; for example, the PROSE approach (Liu et al., 2024b;a; Sun et al., 2024a; Jollie et al., 2024) encodes the PDE directly through symbolic information, which can also be fine-tuned (Sun et al., 2024b). Other models achieve zero- or few-shot generalization by adding PDE structures to the network (Lorsung et al., 2024; Ye et al., 2024); however, these methods require prior knowledge of the PDE structure and information on the data.

In-Context Operator Networks ICONs (Yang et al., 2023a;b; Yang & Osher, 2024) learn operators through observing input-output function pairs, enabling few-shot generalization across various PDEs without explicit PDE representation or fine-tuning. ICONs have demonstrated success in handling forward and inverse problems in ODEs, PDEs, and mean-field control scenarios. However, they face significant computational challenges when processing dense data, as they represent functions through scattered point to-

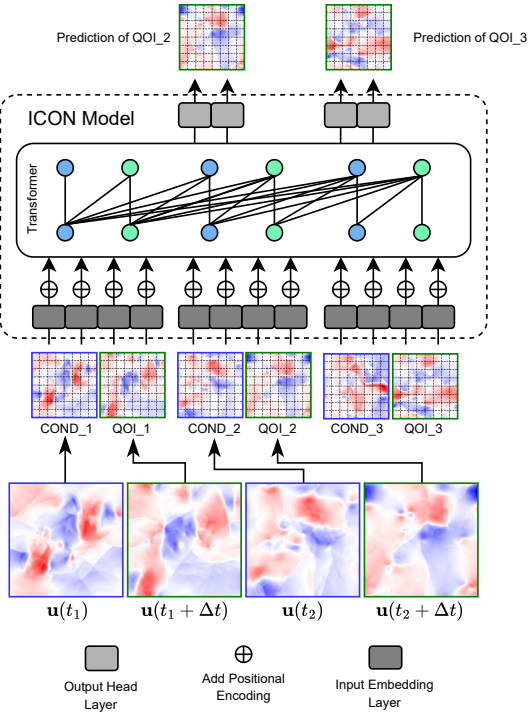


Figure 1. VICON model overview. The inputs to the model are pairs of conditions (COND) and quantities of interest (QOI), which are patchified and flattened before feeding into the transformer layers. The outputs, which represent different patches in the output frame, are transformed back to obtain the final predictions.

kens, resulting in quadratic computational complexity. This limitation has largely restricted their application to 1D problems, with only sparse sampling possible in 2D cases. Our work addresses these limitations by introducing a vision transformer architecture that efficiently handles dense 2D data while maintaining the benefits of in-context operator learning.

3. Preliminaries

ICONS were introduced and developed in (Yang et al., 2023a;b; Yang & Osher, 2024). They adapt the in-context learning framework from large language models, aiming to train a model that can learn operators from prompted function examples.

Denote an ICON model as \mathcal{T}_θ , where \mathcal{T} is a transformer with trainable parameters θ . The model takes input as a sequence comprising I pairs of conditions (c) and quantities of interest (q) as $\{\langle c_i, q_i \rangle\}_{i=1}^I$, where each c, q contains multiple tokens representing a single function. The model outputs tokens representing future functions at the positions

of c , i.e. “next function prediction” similar to “next token predictions” in LLM. More precisely, for $J \in \{1, \dots, I - 1\}$, the model predicts \tilde{q}_{J+1} given the leading J pairs and the condition c_{J+1} :

$$\tilde{q}_{J+1} = \mathcal{T}_\theta[c_{J+1}; \{\langle c_i, q_i \rangle\}_{i=1}^J]. \quad (1)$$

For training parallelization, ICON uses a special causal attention mask to perform autoregressive learning (i.e., the model only sees the leading J pairs and c_{J+1} when predicting \tilde{q}_{J+1}) and enables output of all predictions within a single forward pass (Yang et al., 2023b):

$$\{\tilde{q}_i\}_{i=1}^I = \mathcal{T}_\theta[\{\langle c_i, q_i \rangle\}_{i=1}^I]. \quad (2)$$

Training loss is computed as the mean squared error (MSE) between the predicted states \tilde{q}_i and the ground truth states q_i . To ensure that the model receives sufficient contextual information about the underlying dynamics, we only compute errors for the indices $i > I_{\min}$, effectively requiring at least I_{\min} examples in context. This approach has demonstrated empirical improvements in model performance.

After training, ICON can process a new set of $\langle c, q \rangle$ pairs in the forward pass to in-contextually learn operators and employ them to predict future functions using new conditions:

$$\tilde{q}_k = \mathcal{T}_\theta[c_k; \{\langle c_i, q_i \rangle\}_{i=1}^J], \quad (3)$$

where $J \in \{I_{\min}, \dots, I - 1\}$ is the number of in-context examples, and $k > I$ is the index of the condition for future prediction.

Importantly, all pairs in the same sequence must be formed with the same operator mapping (i.e., the same PDE and consistent timestep size), while operators can vary across different sequences.

4. Methodology

4.1. Problem Setup

We consider the forward problem for multiple time-dependent PDEs that are *temporally homogeneous Markovian*, defined on domain $\Omega \subseteq \mathbb{R}^2$ with solutions represented by $\mathbf{u}(\mathbf{x}, t) : \Omega \times [0, T] \rightarrow \mathbb{R}^c$, where c is the number of channels. Given the initial I_0 frames, $\{\mathbf{u}_i = \mathbf{u}(\cdot, t_i) \mid i = 1, \dots, I_0\}$, the task is to predict future solutions.

Each solution frame is discretized as a three-dimensional tensor $\mathbf{u}_t = \mathbf{u}(\cdot, t) \in \mathbb{R}^{N_x \times N_y \times c}$, where N_x and N_y are the spatial grid sizes. Due to the homogeneous Markovian property, given the initial data $\{\mathbf{u}_t \mid t \geq 0\}$ from some PDE indexed by i_p , for a fixed Δt , there exists an operator $\mathcal{L} = \mathcal{L}_{\Delta t}^{(i_p)}$ that maps frame \mathbf{u}_t to frame $\mathbf{u}_{t+\Delta t}$ for any $t \geq 0$. Our goal is to train a model that learns these operators $\mathcal{L}_{\Delta t}^{(i_p)}$ from the sequence of function pairs generated from

Table 1. (Comparison) Summary of Rollout Rescaled L^2 Error (scale by std) for different methods across various cases. The best results are highlighted in bold.

Rollout Rescaled L^2 Error	Case	Ours (single step)	Ours (multi step)	MPP-B (McCabe et al., 2023)
Step 1 [1e-2]	PDEArena-Incomp	11.10	11.10	6.17
	PDEBench-Comp-LowVis	15.61	15.61	16.37
	PDEBench-Comp-HighVis	5.97	5.97	20.90
Step 5 [1e-2]	PDEArena-Incomp	23.00	25.79	14.81
	PDEBench-Comp-LowVis	24.56	39.54	45.45
	PDEBench-Comp-HighVis	19.73	28.24	42.93
Step 10 [1e-2]	PDEArena-Incomp	36.18	35.02	27.89
	PDEBench-Comp-LowVis	37.47	51.50	64.11
	PDEBench-Comp-HighVis	57.88	79.52	144.46
Last Step [1e-2]	PDEArena-Incomp	77.81	70.83	93.52
	PDEBench-Comp-LowVis	39.03	48.82	65.32
	PDEBench-Comp-HighVis	71.17	96.23	185.29
All average [1e-2]	PDEArena-Incomp	56.27	51.32	55.95
	PDEBench-Comp-LowVis	27.08	36.29	46.68
	PDEBench-Comp-HighVis	30.06	40.06	72.37

the initial frames. Notably, even within the same dataset and fixed Δt , different trajectories can exhibit different dynamics (e.g., due to different Reynolds numbers Re). These variations are implicitly captured by the function pairs in our framework.

4.2. Vision In-Context Operator Networks

The original ICONs represent functions as scattered sample data points, where each data point is projected as a single token. For higher spatial dimensions, this approach necessitates an excessively large number of sample points and extremely long sequences in the transformer. Due to the inherent quadratic complexity of the transformer, this approach becomes computationally infeasible for high-dimensional problems.

Inspired by the Vision Transformer (ViT) (Dosovitskiy, 2020), we address these limitations by dividing the physical fields into patches, where each patch is flattened and projected as a token. This approach, which we call Vision In-Context Operator Networks (VICON), has its forward process illustrated in Figure 1 and detailed in the following.

First, the input \mathbf{u}_t and output $\mathbf{u}_{t+\Delta t}$ functions are divided into patches $\{\mathbf{p}_t \in \mathbb{R}^{R_x \times R_y \times c}\}_{k=1}^{N_c}$ and $\{\mathbf{p}_{t+\Delta t} \in \mathbb{R}^{R_x \times R_y \times c}\}_{l=1}^{N_q}$, where R_x, R_y are the resolution of a patch and N_c and N_q are the number of patches for the input and output functions, respectively. Note that N_q can differ from N_c ; for example, additional boundary paddings in the input functions result in $N_c > N_q$. However, in this work, we maintain $N_c = N_q$ throughout our experiments. For conciseness, we denote the k -th patch of the input function as \mathbf{C}_i^k and the l -th patch of the output function as \mathbf{Q}_i^l , where $i \in \{1, \dots, I\}$ is the pair index. These patches are then projected into a unified d -dimensional latent

embedding space using a shared learnable linear function $f_\phi : R_x \times R_y \times c \rightarrow \mathbb{R}^d$:

$$\begin{aligned}\hat{\mathbf{c}}_i^k &= f_\phi(\mathbf{C}_i^k), \\ \hat{\mathbf{q}}_i^l &= f_\phi(\mathbf{Q}_i^l),\end{aligned}\tag{4}$$

where $k = 1, \dots, N_c$ and $l = 1, \dots, N_q$ index the patches.

We inject two types of learnable positional encoding before feeding the embeddings into the transformer: (1) patch positional encodings to indicate relative patch positions inside the whole domain, denoted as $\mathbf{E}_p \in \mathbb{R}^{N_p \times d}$, where $N_p = \max\{N_c, N_q\}$; (2) function positional encodings to indicate whether a patch belongs to a input function (using $\mathbf{E}_c \in \mathbb{R}^{I \times d}$) or output function (using $\mathbf{E}_q \in \mathbb{R}^{I \times d}$), as well as their indices in the sequence:

$$\begin{aligned}\mathbf{c}_i^k &= \hat{\mathbf{c}}_i^k + \mathbf{E}_p(k) + \mathbf{E}_c(i), \\ \mathbf{q}_i^l &= \hat{\mathbf{q}}_i^l + \mathbf{E}_p(l) + \mathbf{E}_q(i).\end{aligned}\tag{5}$$

The embeddings \mathbf{c}_i^k and \mathbf{q}_i^l are then concatenated to form the input sequence for the transformer:

$$\mathbf{c}_1^1 \dots \mathbf{c}_1^{N_c}, \mathbf{q}_1^1 \dots \mathbf{q}_1^{N_q}, \dots, \mathbf{c}_I^1 \dots \mathbf{c}_I^{N_c}, \mathbf{q}_I^1 \dots \mathbf{q}_I^{N_q}.$$

To support autoregressive prediction similar to Eq 2, VICON employs an alternating-sized (in the case when $N_c \neq N_q$) block-causal attention mask, as opposed to the conventional triangular causal attention mask as in mainstream generative large language models (Brown et al., 2020) and large vision models (Bai et al., 2024). The mask is defined as follows:

Table 2. Summary of Rollout Rescaled L^2 Error Metrics (single step, scale by std) for Ablation Studies across various cases. The best results are highlighted in bold. Columns: N = PDEArena-Incomp only, E = PDEBench-Comp-LowVis only, C = PDEBench-Comp-HighVis only, N + E = PDEArena-Incomp + PDEBench-Comp-LowVis, N + C = PDEArena-Incomp + PDEBench-Comp-HighVis, E + C = PDEBench-Comp-LowVis + PDEBench-Comp-HighVis. All ablation groups have equal dataset batch sizes, adding up to 30. The best results are marked with **bold**, 2nd-best with underline.

Rollout Rescaled L^2 Error [1e-2]	Case	All included	N	E	C	N + E	N + C	E + C
Step 1	PDEArena-Incomp	11.10	6.72	54.26	58.63	11.11	<u>7.84</u>	53.69
	PDEBench-Comp-LowVis	15.61	88.24	13.59	46.66	14.86	45.96	<u>14.08</u>
	PDEBench-Comp-HighVis	5.79	216.50	49.68	2.81	38.38	<u>3.28</u>	4.65
Step 5	PDEArena-Incomp	23.00	14.18	105.19	85.76	22.95	<u>16.25</u>	100.12
	PDEBench-Comp-LowVis	24.56	106.29	22.10	67.27	23.88	68.15	<u>22.53</u>
	PDEBench-Comp-HighVis	19.73	584.50	170.20	7.49	141.22	<u>8.38</u>	12.41
Step 10	PDEArena-Incomp	36.18	23.90	147.28	110.04	36.87	<u>26.63</u>	142.60
	PDEBench-Comp-LowVis	37.47	119.84	33.94	81.27	36.43	<u>82.27</u>	<u>34.49</u>
	PDEBench-Comp-HighVis	57.88	1903.41	514.88	20.26	410.23	<u>22.65</u>	33.42
Last Step	PDEArena-Incomp	77.81	69.95	289.62	185.34	86.16	<u>71.90</u>	294.19
	PDEBench-Comp-LowVis	39.03	120.74	35.16	83.24	37.61	84.19	<u>35.70</u>
	PDEBench-Comp-HighVis	71.17	2356.29	626.60	24.35	495.02	<u>27.28</u>	40.24
All average	PDEArena-Incomp	56.26	44.35	202.67	142.00	59.65	<u>47.31</u>	203.04
	PDEBench-Comp-LowVis	27.08	107.54	24.35	68.71	26.27	69.52	<u>24.83</u>
	PDEBench-Comp-HighVis	30.06	956.67	263.98	10.91	213.34	<u>12.25</u>	18.06

$$\mathbf{M} = \begin{bmatrix} \mathbf{1}_{N_c, N_c} & \mathbf{0} & \cdots & \mathbf{0} & \mathbf{0} \\ \mathbf{1}_{N_q, N_c} & \mathbf{1}_{N_q, N_q} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{1}_{N_c, N_c} & \mathbf{1}_{N_c, N_q} & \cdots & \mathbf{1}_{N_c, N_c} & \mathbf{0} \\ \mathbf{1}_{N_q, N_c} & \mathbf{1}_{N_q, N_q} & \cdots & \mathbf{1}_{N_q, N_c} & \mathbf{1}_{N_q, N_q} \end{bmatrix} \quad (6)$$

where $\mathbf{1}_{m \times n}$ denotes an all-ones matrix of dimension $m \times n$, and $\mathbf{0}$ represents a zero matrix of the corresponding dimensions.

After obtaining the output tokens from Eq 2, we extract the tokens corresponding to the input patches $\mathbf{c}_i^1, \dots, \mathbf{c}_i^{N_c}$ and denote them as $\tilde{\mathbf{q}}_i^1, \dots, \tilde{\mathbf{q}}_i^{N_c}$. These tokens are then projected back to the original physical space using a shared learnable linear function $g_\psi : \mathbb{R}^d \rightarrow \mathbb{R}^{R_x \times R_y \times c}$:

$$\tilde{\mathbf{Q}}_i^j = g_\psi(\tilde{\mathbf{q}}_i^j), \quad (7)$$

which predicts \mathbf{Q}_i^j .

Throughout our experiments, we set $I_{\min} = 5$ in-context example pairs, providing at least five examples to ensure sufficient information about the dynamics before evaluating errors.

4.3. Prompt Normalization

To address the varying scales across different channels and prompts (i.e. sequences of pairs $\{(\mathbf{c}_i, \mathbf{q}_i)\}_{i=1}^I$), we normalize the data before feeding them into the model. A crucial requirement is to consistently normalize functions within the same sequence, to ensure that the same operator is learned. Denoting the normalization operators for \mathbf{c} and \mathbf{q} as \mathcal{N}_c and \mathcal{N}_q respectively, and the operator in the original space as \mathcal{L} , the operator in the normalized space \mathcal{L}' follows:

$$\mathcal{L}'(\mathcal{N}_c(\mathbf{u})) = \mathcal{N}_q(\mathcal{L}(\mathbf{u})), \quad (8)$$

or equivalently:

$$\mathcal{L}'(\mathbf{v}) = \mathcal{N}_q(\mathcal{L}(\mathcal{N}_c^{-1}(\mathbf{v}))).$$

In this work, we simply set $\mathcal{N}_c = \mathcal{N}_q$, mainly because our maximum timestep stride $s_{\max} = 5$ is relatively small (i.e., the scale distribution does not change dramatically). Specifically, we computed the channel-wise mean μ and standard deviation σ of $\{\mathbf{c}_i\}_{i=1}^I$ in each prompt, then used these values to normalize $\{(\mathbf{c}_i, \mathbf{q}_i)\}_{i=1}^I$ in that prompt. To avoid division by zero, we set a minimum threshold of 10^{-4} for the standard deviation.

4.4. Datasets and Data Augmentation

The dataset used in this work contains three types of equation modeling different fluid dynamics:

1. PDEArena-Incomp (Gupta & Brandstetter, 2022), the incompressible Navier-Stokes (NS) equation
2. PDEBench-Comp-HighVis (Takamoto et al., 2022), the compressible NS equation
3. PDEBench-Comp-LowVis (Takamoto et al., 2022), the compressible NS equation with very low viscosity

Detailed descriptions of the datasets are provided in Appendix A.

The temporally homogeneous Markovian property (Section 4.1) enables natural data augmentation for training and flexible rollout strategy for inference (Section 4.5): by striding with larger timesteps to reduce the number of autoregressive steps in long-term prediction tasks: $\Delta t = s \Delta \tau_{i_p} \mid s = 1, 2, \dots, s_{\max}$, where τ_{i_p} is the timestep size for recording

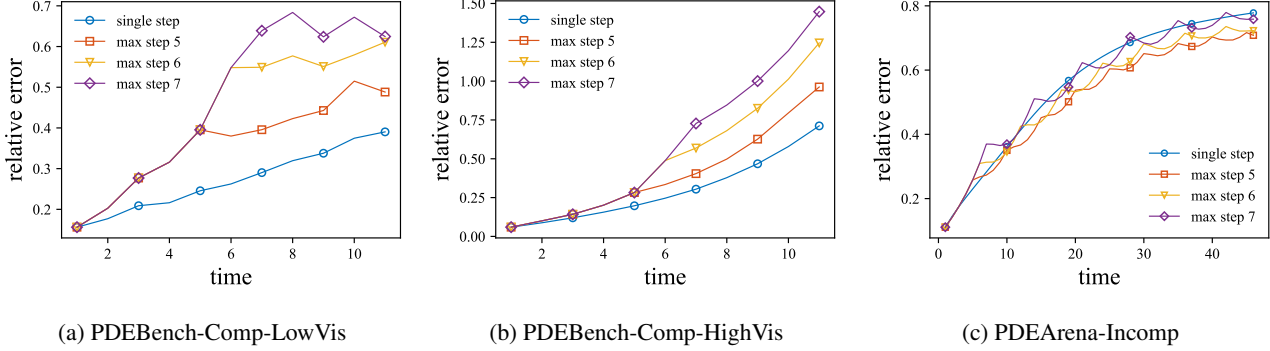


Figure 2. Comparison of rollout errors across different datasets, using single-step and multi-step strategies with varying maximum step sizes ($s_{\max} = 5, 6, 7$).

in trajectory i_p . During training, for each trajectory in the dataset, we first sample a stride size $s \sim \mathcal{U}\{1, \dots, s_{\max}\}$, then form $\langle \mathbf{u}_t, \mathbf{u}_{t+s\Delta\tau} \rangle$ pairs for the corresponding operator.

4.5. Inference with Flexible Strategies

As mentioned in Section 4.4, VICON can make predictions with different timestep strides. Specifically, given I_0 initial frames $\{\mathbf{u}_i\}_{i=1}^{I_0}$, we can form in-context example pairs $\{\langle \mathbf{u}_i, \mathbf{u}_{i+s} \rangle\}_{i=1}^{I_0-s}$. Then for $j \geq 1$, we can set the question condition (c_k in Equation 3) as \mathbf{u}_j to predict \mathbf{u}_{j+s} , i.e. s -step prediction.

To make predictions for long-term rollout, we can use VICON in an autoregressive fashion. The flexibility in predicting with different strides enables various rollout strategies. We explore two natural approaches: single-step rollout and multi-step rollout that advances in large strides to reduce the number of rollout steps.

For single-step rollout, we fix the in-context examples as $\{\langle \mathbf{u}_i, \mathbf{u}_{i+1} \rangle\}_{i=1}^{I_0-1}$, and call the model to make prediction of \mathbf{u}_{I_0+1} with \mathbf{u}_{I_0} as the question condition. Then we can predict \mathbf{u}_{I_0+2} with the predicted \mathbf{u}_{I_0+1} as the question condition. So on and so forth. Multi-step rollout is more complicated: suppose the maximum prediction stride is s_{\max} , we first make s -step predictions with \mathbf{u}_{I_0} as the question condition, to make prediction of $\{\mathbf{u}_{I_0+s}\}_{s=1}^{s_{\max}}$. Then with the predicted $\{\mathbf{u}_{I_0+s}\}_{s=1}^{s_{\max}}$ as the question conditions, we make s_{\max} -step predictions to get $\{\mathbf{u}_{I_0+s_{\max}+s}\}_{s=1}^{s_{\max}}$. So on and so forth. Such multi-step rollout strategy is the same as the one in (Yang & Osher, 2024). Examples of rollout strategies are provided in the Appendix B.3.

4.6. Evaluation Metric

We evaluate the rollout accuracy of VICON using two different strategies described in Section 4.5. The evaluation uses rescaled and absolute L^2 errors between the predicted

and ground truth frames, starting from the frame $I_0 + 1$. For relative scaling coefficients, we use channel-wise scaling standard deviation σ of ground truth frames, which vary between different prompts.

5. Experimental Results

The detailed experimental results are presented in Table 1 and Table 8, where we report the scaled and absolute L^2 roll-out errors for the three datasets. We record the rollout error at time steps 1, 5, 10, and the last step, along with the average error across all time steps. Due to the flexibility of time step strides, a single trained model adopts two rollout strategies: single-step and multi-step (Section 4.5) during inference, and we report results for both strategies.

Superior Accuracy for Long-term Predictions We compare our model with MPP (McCabe et al., 2023), which is trained as a single-step predictor that predicts the next frame given the previous frames with a fixed Δt . We observe that VICON (both strategies) outperforms MPP at all timesteps for the PDEBench-Comp-LowVis and PDEBench-Comp-HighVis datasets. For longer rollouts, VICON shows a significant reduction in scaled L^2 error of 40% and 61.6% for PDEBench-Comp-LowVis and PDEBench-Comp-HighVis datasets, respectively. For the PDEArena-Incomp dataset, while VICON slightly underperforms MPP initially, it achieves better results for longer-step rollouts (the last step and the averaged error across the whole time horizon of 46 steps). In general, our method demonstrates superior long-term predictions compared to MPP.

Flexible Rollout Strategy To demonstrate the flexibility of our model rollout strategies (Section 4.5), we evaluate the performance of single-step and multi-step rollout strategies with varying maximum step sizes ($s_{\max} = 5, 6, 7$) in Figure 2. We observe that the model performs best with

single-step rollout for PDEBench-Comp-LowVis and PDEBench-Comp-HighVis, while multi-step rollout with $s_{\max} = 5$ shows the best performance for PDEArena-Incomp.

This performance difference can be attributed to two key factors: (1) PDEBench-Comp-LowVis and PDEBench-Comp-HighVis employ significantly larger recording timestep sizes with fewer total frames (detailed in Section A), making operator learning more challenging. In such cases, the error in a single large step outweighs the potential benefits of reduced autoregressive rollout numbers. (2) Conversely, PDEArena-Incomp contains longer trajectories with smaller recording timesteps, making operator learning easier, where larger strides reduce the number of rollouts and thus error accumulation.

This flexibility in inference allows the model to adapt to different datasets and prediction tasks without retraining, making VICON more versatile for various task settings (such as climate prediction at short and long terms).

Stride Generalization When evaluated with unseen strides ($s_{\max} = 6, 7$), our model maintains reasonable performance and even outperforms the single-step strategy for PDEArena-Incomp. This capability stems from ICON’s ability to implicitly learn operators via context pairs. When applied to experimental measurements, this ability to extrapolate and generalize to unseen timestep sizes provides greater tolerance for sampling rates, which can be constrained by hardware limitations.

Computational Efficiency In Table 3, we compare the computational resources required by VICON and MPP. Our model demonstrates greater efficiency, requiring approximately one third of the inference time per frame and using 75% of the total parameters compared to MPP.

Impact of Data Diversity on Generalization To evaluate the generalization capabilities of the model across datasets, we performed ablation studies by training on various combinations of datasets. The results are presented in Table 2.

The experiments reveal that the model achieves optimal performance on individual datasets when trained exclusively on them, though at the cost of poor accuracy on others. As more datasets are incorporated during training, a trade-off emerges: Performance on individual datasets slightly decreases, but the model learns to handle different simulation types simultaneously.

These results demonstrate that VICON is a generalized few-shot model for diverse fluid dynamics problems, suggesting potential extensions through additional datasets and pretrain-finetune approaches to address a much wider range of settings.

Table 3. (Comparison) Summary of Resource and Timing Metrics for different methods.

Resource and Timing Metrics	Ours	MPP-B (McCabe et al., 2023)
Training cost [GPU hrs]	58	64
Rollout time per step [ms]	8.7	25.7
Model Param Size	88M	116M

6. Conclusion and Future Works

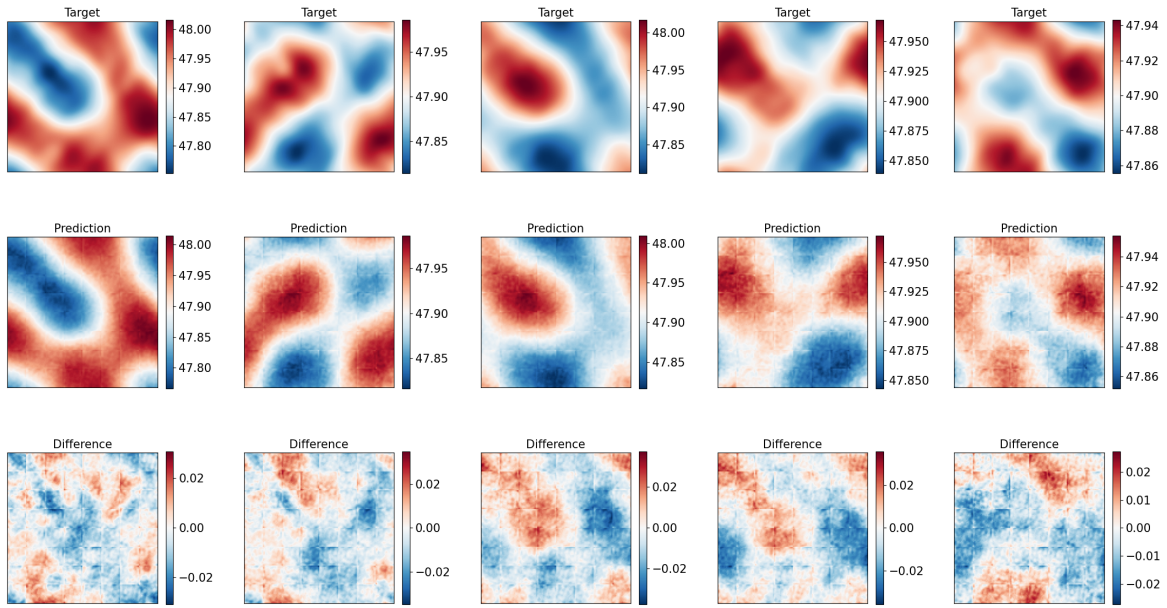
We present VICON, a vision-transformer-based in-context operator network that efficiently handles two-dimensional fluid dynamics problems. By processing dense physical fields through patch-wise operations, VICON overcomes the computational limitations of naive ICON approaches while maintaining their flexibility in multi-physics operator learning.

In extensive experiments, we have demonstrated that VICON achieves superior performance compared to existing MPP, especially in long-term predictions. VICON also offers flexible rollout strategies with varying time-step strides, allowing users to adapt predictions based on specific scenario requirements.

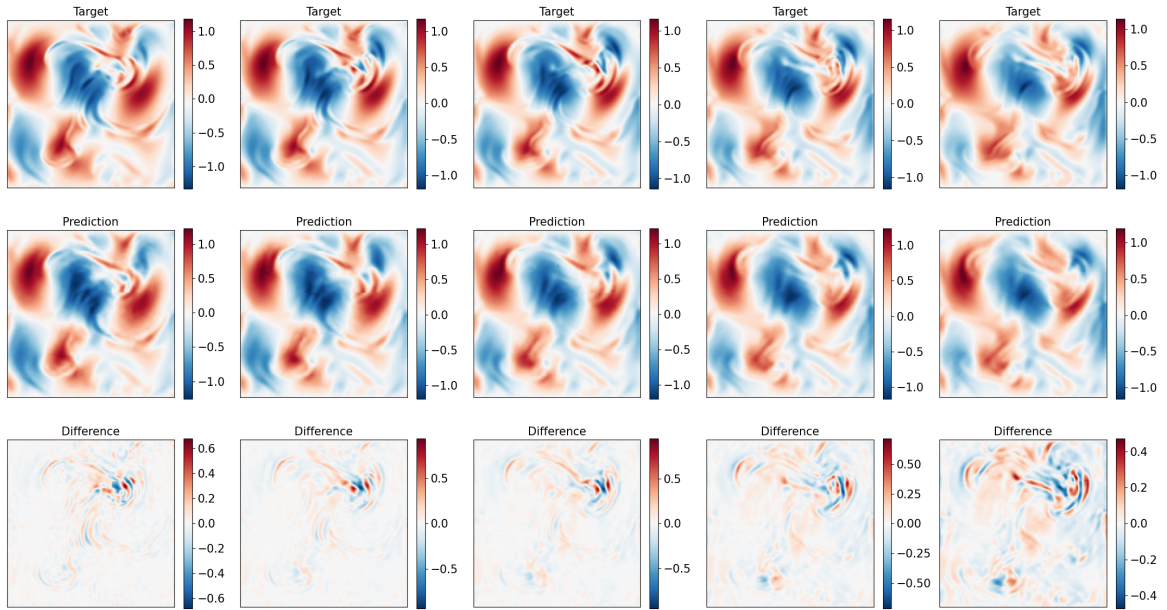
Several promising directions remain for future investigation. Our method currently requires full domain input. 1. Incorporating physical priors, such as translational invariance, could enable domain decomposition for training on smaller sub-domains, effectively decoupling computational complexity from domain size. 2. Extending VICON to handle irregular domains and different data structures such as graphs or point clouds would broaden its applicability to more realistic scenarios. 3. Our current approach of joining channels across datasets has limited scalability. Drawing inspiration from machine translation, where subword tokenization enables handling multiple languages without a fixed vocabulary union, developing similar adaptive approaches for physical field representations could offer a more scalable solution.

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(a) 5 output steps for PDEBench-Comp-HighVis dataset. The channel plotted is the pressure field in equation 12. Each column represents a different timestamp.



(b) 5 output steps for PDEArena-Incompdataset. The channel plotted is the x-velocity in equation 9, i.e. u_x . Each column represents a different timestamp.

Figure 3. Two example outputs for the VICON model.

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A. Dataset Details

A.1. PDEArena-Incomp Dataset

The incompressible Navier-Stokes dataset comes from PDEArena (Gupta & Brandstetter, 2022). The data are generated from the equation

$$\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u} = -\nabla p + \mu \Delta \mathbf{u} + \mathbf{f}, \quad (9)$$

$$\nabla \cdot \mathbf{u} = 0. \quad (10)$$

The space-time domain is $[0, 32]^2 \times [18, 105]$ where $dt = 1.5$ and the space resolution is 128×128 . A scalar particle field is being transported with the fluids. The velocity fields satisfy Dirichlet boundary condition, and the scalar field satisfies Neumann boundary condition. The forcing term \mathbf{f} is randomly sampled. The quantities of interest are the velocities and the scalar particle field.

A.2. PDEBench-Comp-HighVis and PDEBench-Comp-LowVis Datasets

The PDEBench-Comp-HighVis and PDEBench-Comp-LowVis datasets come from PDEBench Compressible Navier-Stokes dataset (Takamoto et al., 2022). The data are generated from the equation

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{u}) = 0, \quad (11)$$

$$\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u}) = -\nabla p + \eta \Delta \mathbf{u} + (\zeta + \eta/3)\nabla(\nabla \cdot \mathbf{u}), \quad (12)$$

$$\partial_t \left(\varepsilon + \frac{\rho u^2}{2} \right) = -\nabla \cdot \left(\left(\varepsilon + p + \frac{\rho u^2}{2} \right) \mathbf{u} - \mathbf{u} \cdot \sigma' \right). \quad (13)$$

The space-time domain is $\mathbb{T}^2 \times [0, 1]$ where $dt = 0.05$. The datasets contain different combinations of shear and bulk viscosities. We group the ones with larger viscosities into the PDEBench-Comp-HighVis dataset, and the ones with extremely small (1e-8) viscosities into the PDEBench-Comp-LowVis dataset. The PDEBench-Comp-HighVis dataset has space resolution 128×128 . The PDEBench-Comp-LowVis dataset has raw space resolution 512×512 and is downsampled to 128×128 through average pooling for consistency. The quantities of interest are velocities, pressure, and density.

A.3. QOI Union and Channel Mask

Since each dataset contains different sets of quantities of interest, we take their union to create a unified representation. The unified physical field has 7 channels in total, with the following ordering:

1. Density (ρ)
2. Velocity at x direction (u_x)
3. Velocity at y direction (u_y)
4. Pressure (P)
5. Vorticity (ω)
6. Passively transported scalar field (S)
7. Node type indicator (0: interior node, 1: boundary node)

For each dataset, we use a channel mask to indicate its valid fields and only calculate loss on these channels. The node type channel is universally excluded from loss calculations across all datasets.

B. Experiment Details

B.1. VICON Model Details

Here we provide key architectural parameters of VICON model implementation in Table 4.

Table 4. Model Configuration Details

Key Hyperparameters & Shapes	
<i>Patch Configuration</i>	
Input patch dimension	8×8
Output patch dimension	8×8
Patch resolution	16×16
<i>Positional Encodings Shapes</i>	
Patch positional encodings	[64, 1024]
Function positional encodings	[20, 1024]
<i>Transformer Configuration</i>	
Hidden dimension	1024
Number of attention heads	8
Feedforward dimension	2048
Number of layers	10
Dropout rate	0.0
Number of COND & QOI pairs	10

B.2. Training Details

We implement our method in PyTorch (Paszke et al., 2019) and utilize data parallel training (Li et al., 2020) across two NVIDIA RTX 4090 GPUs. We employ the AdamW optimizer with a cosine learning rate schedule that includes a linear warmup phase. We apply gradient clipping with a maximum norm of 1.0 to ensure training stability. All optimization parameters are detailed in Table 5.

Table 5. Optimization Hyperparameters

Parameter	Value
<i>Learning Rate Schedule</i>	
Scheduler	Cosine Annealing with Linear Warmup
Peak learning rate	1×10^{-4}
Final learning rate	1×10^{-7}
Warmup steps	20,000
Total steps	200,000
<i>Optimization Settings</i>	
Optimizer	AdamW
Weight decay	1×10^{-4}
Gradient norm clip	1.0

B.3. Rollout Strategy Example

We demonstrate two rollout strategies in Tables 6 and 7, showing single-step and multi-step strategies, respectively. In both cases, the initial frames span from time step 0 to 9, and we aim to predict the trajectory up to time step 20.

Table 6. Single-step Rollout Strategy Example

Rollout index	Example CONDs	Example QOIs	Question COND	Predict QOI
1	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	9	10
2	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	10	11
3	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	11	12
4	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	12	13
5	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	13	14
6	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	14	15
7	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	15	16
8	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	16	17
9	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	17	18
10	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	18	19
11	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	19	20

As shown in Table 7, the multi-step strategy initially uses smaller strides to build sufficient examples, then employs maximum strides for later rollouts, as detailed in Section 4.5. We note that repeated in-context examples appear in Table 7, which is common when the maximum stride is large and the initial frames cannot form enough examples. While the number of in-context examples in VICON is flexible and our model can accommodate fewer examples than the designed length, our

preliminary experiments indicate that the model performs better with more in-context examples, even when some examples are repeated.

Table 7. Multi-step Rollout Strategy Example ($s_{\max} = 5$)

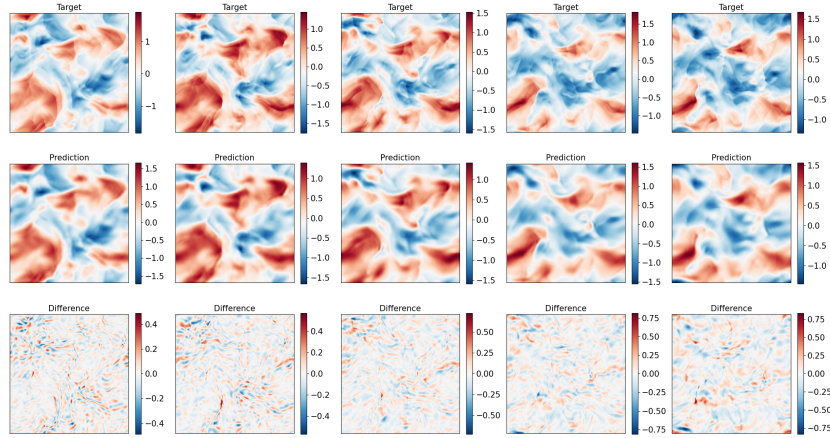
Rollout index	Example CONDS	Example QOIs	Question COND	Predict QOI
1	[0,1,2,3,4,5,6,7,8]	[1,2,3,4,5,6,7,8,9]	9	10
2	[0,0,1,2,3,4,5,6,7]	[2,2,3,4,5,6,7,8,9]	9	11
3	[0,0,1,1,2,3,4,5,6]	[3,3,4,4,5,6,7,8,9]	9	12
4	[0,0,1,1,2,2,3,4,5]	[4,4,5,5,6,6,7,8,9]	9	13
5	[0,0,1,1,2,2,3,3,4]	[5,5,6,6,7,7,8,8,9]	9	14
6	[0,0,1,1,2,2,3,3,4]	[5,5,6,6,7,7,8,8,9]	10	15
7	[0,0,1,1,2,2,3,3,4]	[5,5,6,6,7,7,8,8,9]	11	16
8	[0,0,1,1,2,2,3,3,4]	[5,5,6,6,7,7,8,8,9]	12	17
9	[0,0,1,1,2,2,3,3,4]	[5,5,6,6,7,7,8,8,9]	13	18
10	[0,0,1,1,2,2,3,3,4]	[5,5,6,6,7,7,8,8,9]	14	19
11	[0,0,1,1,2,2,3,3,4]	[5,5,6,6,7,7,8,8,9]	15	20

B.4. MPP (McCabe et al., 2023) Details

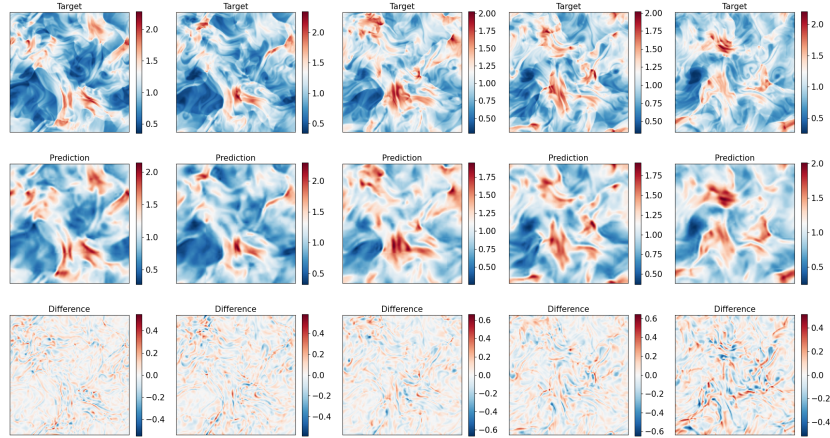
For a fair comparison, we retrained MPP-B using our training dataset using the same model configurations and optimizer hyperparameters. We evaluate MPP-B using the same testing setup and metric.

C. More Results and Visualizations

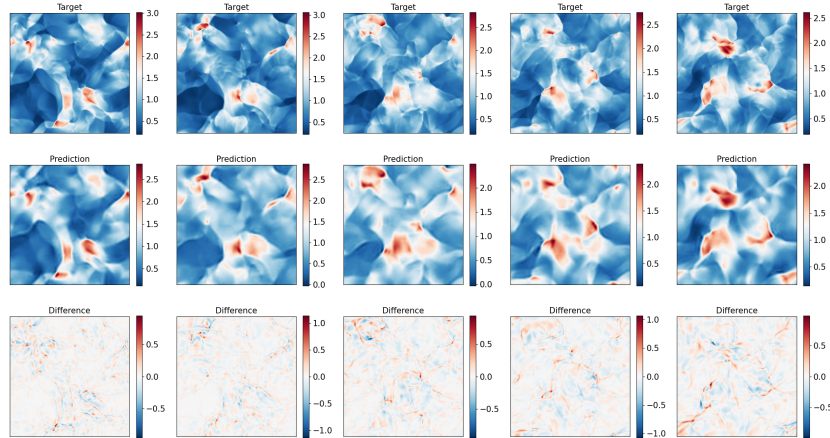
Table 8 summarizes the rollout L^2 errors, and Figure 4 presents additional visualizations of the VICON model outputs.



(a) The channel plotted is the y-velocity.



(b) The channel plotted is the density field.



(c) The channel plotted is the pressure field.

Figure 4. Example outputs for the VICON model. 5 output steps for PDEBench-Comp-LowVis dataset. Each column represents a different timestep.

Table 8. (Comparison) Summary of Rollout Absolute L^2 Error for different methods across various cases. The best results are highlighted in bold.

Rollout L^2 Error	Case	Ours (single step)	Ours (multi step)	MPP (McCabe et al., 2023)
Step 1 [1e-2]	PDEArena-Incomp	5.67	5.67	3.12
	PDEBench-Comp-LowVis	21.97	21.97	24.47
	PDEBench-Comp-HighVis	1.55	1.55	4.73
Step 5 [1e-2]	PDEArena-Incomp	10.51	11.83	6.79
	PDEBench-Comp-LowVis	31.46	56.80	57.50
	PDEBench-Comp-HighVis	2.50	6.21	6.52
Step 10 [1e-2]	PDEArena-Incomp	14.82	14.36	11.49
	PDEBench-Comp-LowVis	45.91	65.60	78.54
	PDEBench-Comp-HighVis	3.39	6.49	8.98
Last Step [1e-2]	PDEArena-Incomp	16.82	15.12	19.47
	PDEBench-Comp-LowVis	48.98	63.94	85.60
	PDEBench-Comp-HighVis	3.58	6.24	9.56
All average [1e-2]	PDEArena-Incomp	16.64	15.18	15.77
	PDEBench-Comp-LowVis	34.73	50.21	60.68
	PDEBench-Comp-HighVis	2.64	4.85	7.04