

ALGEBRA QUALIFYING EXAM

SEPTEMBER 15, 2026

Test instructions:

Write your UCLA ID number on the upper right corner of *each* sheet of paper you use. Do not write your name anywhere on the exam. Please staple your problems in the order they are listed in the exam.

No books, notes, phones, or other printed or electronic materials can be used.

Throughout the exam, *justify your answers*.

Complete 8 of the 10 problems. Clearly indicate, by circling the numbers below, which 8 problems should be graded:

1	2	3	4	5	6	7	8	9	10
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Problem 1. Let $\mathbb{F}$ be a finite field with q elements. Let N be the number of endomorphisms of $\mathbb{F}$ as an abelian group. Compute $q^{1/\log_q(N)}$ in terms of $\mathbb{F}$.

Problem 2. Let k be a field, and let $f: A \rightarrow B$ be a homomorphism of finitely generated commutative k -algebras. Show that for every maximal ideal $\mathfrak{m} \subset B$, the ideal $f^{-1}(\mathfrak{m})$ is maximal in A .

Problem 3. Let k be a field of characteristic not 2 and $P \in k[X]$ a polynomial such that $P(X^2) = (P(X+1))^2$. Prove that $P = 0$ or 1. [Hint: Count the distinct roots.]

Problem 4. Consider two short exact sequences of A -modules $0 \rightarrow L_i \rightarrow M_i \rightarrow N_i \rightarrow 0$ for $i = 1, 2$. For another ring B , let $F: A\text{-Mod} \rightarrow B\text{-Mod}$ be an exact faithful functor. If the images of the sequences are isomorphic in $B\text{-Mod}$, can we conclude that they are isomorphic in $A\text{-Mod}$? Prove it or give an explicit counterexample.

Problem 5. Let M be a finitely generated module over a ring A , and let $f: M \rightarrow M$ be a surjective A -linear homomorphism.

- (a) Show that if A is commutative then f is an isomorphism.
- (b) Show that this can fail if A is not commutative.

Problem 6. Let p be any prime number, and let l be any prime number not equal to 2. Write $\mathbb{F}_p$ for the field of order p . Let $PGL(2, \mathbb{F}_p)$ be the group of invertible 2×2 matrices over $\mathbb{F}_p$ modulo the subgroup of scalar matrices. The problem is to find a Sylow l -subgroup of $PGL(2, \mathbb{F}_p)$. Also, describe the structure of that subgroup as well as you can.

Problem 7. Let $F: \text{CommRings} \rightarrow \text{Rings}$ be the functor that takes a commutative ring R to R in the category of all rings. Does F have a left adjoint? Does F have a right adjoint? Justify your answers. (If the answer is yes, describe the adjoint functor explicitly.)

Problem 8. Show that there is no simple group of order 30.

Problem 9. Let G be the infinite dihedral group, $G = \langle x, y \mid yxy^{-1} = x^{-1}, y^2 = 1 \rangle$. Show that the Jacobson radical of the group algebra $\mathbb{C}G$ is zero.

Problem 10. Let M be a complex of free abelian groups, written as

$$\cdots \rightarrow M_{i+1} \rightarrow M_i \rightarrow M_{i-1} \rightarrow \cdots$$

Let n be a positive integer. Write $H_i(M)$ for the i th homology group of M . Construct a surjective homomorphism from $H_i(M/n)$ to $H_{i-1}(M)[n]$. (For an abelian group A , A/n means A/nA , and $A[n]$ means the n -torsion subgroup $\{x \in A : nx = 0\}$. For a complex M , M/n means the complex $\cdots \rightarrow M_i/n \rightarrow \cdots$.)